FARE EVASION IN PUBLIC TRANSPORT: HOW DOES IT AFFECT THE OPTIMAL DESIGN AND PRICING?

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ABSTRACT

Fare evasion produces significant revenue losses in public transport systems. Recent research has found that low service quality and high prices are important determinants of fare evasion. However, the economic literature that studies the optimal public transport provision has overlooked the phenomenon. We develop a demand model of horizontal differentiation to investigate how fare evasion affects the design and pricing of public transport that maximizes utilitarian social welfare. We show analytically that fare evasion can create incentives to reduce public transport prices and improve service quality, putting upward pressure on its subsidization. We perform numerical simulations and sensitivity analysis to quantify the impact of these incentives on the public transport provision. These simulations confirm that the incentive to reduce fare evasion can lead to an optimal design and price that requires subsidies.

1. INTRODUCTION

Fare evasion in public transport is a major concern for governments worldwide because it produces millions of dollars in lost revenues that risk the systems' financial health (Bonfanti & Wagenknecht, 2010). For example, 30% of bus users in Santiago, Chile, and Bogota, Colombia, do not pay the fare, producing annual losses estimated at USD 140 million (Porath & Galilea, 2020; Probogotá, 2022). On the other hand, evasion rates are usually below 5% in the developed world. However, some independent studies suggest that official reports may be underestimating the problem. For example, recent studies have found double-digit evasion rates in cities like Cagliari and Reggio Emilia, Italy (Barabino et al., 2022; Bucciol et al., 2013), Melbourne (Currie & Delbosc, 2017), Lyon (Egu & Bonnel, 2020), and Athens (Milioti et al., 2020).

In line with the increasing concern of transport authorities, the number of studies focused on understanding why people evade payment has grown exponentially in recent years (see, Barabino et al., 2020, for a recent review). These studies show that fare evasion depends on factors such as the users' characteristics and attitudes, the deterrence and enforcement measures implemented by authorities, and the contractual incentives to public transport operators and drivers (Dai et al., 2018; Clarke et al., 2010; Ramírez et al., 2022; Delbosc & Currie, 2019; Tamblay et al., 2017). Perhaps more significant for public transport authorities is that low service quality and high prices are essential determinants of fare dodging (Busco et al., 2021; Barabino et al., 2015; Guzman et al., 2021; Cools et al., 2018). This occurs for two reasons: First, evaders are usually dissatisfied with the price and overall level of service (Delbosc & Currie, 2016a,b; González et al., 2019). Second, conditions like crowding at bus stops and vehicles favor evasion because it increases the anonymity perception and the so-called contagion effect, which makes dodging easier (Cantillo et al., 2022; Currie & Delbosc, 2017; Salis et al., 2018). Consequently, recent studies have recommended increasing the level of service and the subsidies to reduce the number of evaders in public transport (see, e.g., Porath & Galilea, 2020; Barabino et al., 2022; Milioti et al., 2020). Nevertheless, the economic efficiency of this proposal and its consequences for operators' and users' surplus have not been studied.

The transport economic literature has investigated in detail the welfare-maximizing public transport provision (see Hörcher & Tirachini, 2021, for a recent review). However, this strand of the literature disregards evasion or considers it an exogenous variable. On the other hand, the economic literature on fare evasion has focused on inspection and fines, ignoring the effect of quality and prices on the evading behavior (see, e.g., Barabino & Salis, 2019). This paper aims to fill these gaps by acknowledging that fare dodging is partially determined by the decisions of transport authorities regarding public transport quality, price, and inspection rate. Thus, our objective is to investigate the effect of fare evasion on the welfare-maximizing design and pricing of public transport.

We develop a theoretical model where a mass of potential users chooses between paying or evading when using an isolated bus line provided by a public operator. Individuals base their decision on the service quality, fare, and inspection rate of public transport. Analytical expressions show that fare evasion creates incentives to reduce public transport fares below the standard marginal-cost pricing, putting upward pressure on subsidies. Regarding service quality, evasion creates an incentive to improve the frequency if reducing the inspection rate raises inspection profits, which may occur if the inspection is too costly or if revenues from fines are negligible.

We illustrate our theoretical model with numerical simulations aiming to reflect a representative public transport system with a 30% evasion rate. Our results show that the evasion rate in this baseline scenario is ten percentage points above optimal, so the social planner increases inspection to reduce evasion and raise revenues from fines. Consequently, the cost recovery ratio rises from 60% in the baseline scenario to almost 80% in the optimal solution. Nevertheless, and unlike the previous literature, the optimal design and pricing of public transport considering fare evasion require subsidies for realistic values of the cost of public funds (Basso & Silva, 2014; Börjesson et al., 2019; Hörcher et al., 2020). This confirms that the Mohring effect (1972; 1976) and the incentive to reduce fare evasion can countervail the distortions in the markets where the government collects taxes, showing that the relationship between fare evasion and the price and quality of public transport strongly affects its optimal provision.

The rest of the paper is structured as follows. Section 2 presents the model. Section 3 describes and interprets the analytical solution. Section 4 shows a numerical implementation of the model, and Section 5 concludes.

2. THE MODEL

We consider a bus line that serves a mass of individuals distributed homogeneously along a corridor, as in the seminal works by Mohring (1972, 1976) and Jansson (1980, 1984). This simple and tractable approach allows us to obtain transparent insights into the impact of fare evasion in public transport provision and compare our results with the well-known pricing and quality rules obtained in previous studies. Following fare evasion literature (Buehler et al., 2017; Besfamille et al., 2022; E. C. Silva & Kahn, 1993), we model individual behavior as a discrete choice between (i) traveling by bus paying the fare (formal users or non-evaders), (ii) traveling by bus evading the payment and assuming the risk of being inspected and fined (informal users or evaders), and (iii) not traveling (leave the market).

A public operator provides the bus service while a public agency performs the inspection activities. A social planner defines the bus line frequency f, the fare τ, and the probability π of being inspected and fined (i.e., the inspection rate). For the sake of simplicity, we assume that the bus capacity is fixed and sufficiently high for the bus occupancy to be below the maximum possible level at an interior solution.

2.1. Demand model

Let us consider a benefit function $B(Y^p, Y^e)$, where Y^p and Y^e denote the number of formal and informal users. Ignoring income effects, the benefit function represents the consumers' benefit as their total willingness to travel for a particular combination $\{Y^p, Y^e\}$ (see, e..g., Kraus, 2003; Pressman, 1970; Tirachini & Hensher, 2012). Therefore, it is a generalization of the area under the inverse demand function (Small & Verhoef, 2007, pp. 155-159). Then, the marginal benefit from traveling formally or informally, which can be interpreted as an inverse demand function, is given by:

$$B_i = \frac{\partial B(Y^p, Y^e)}{\partial Y^i},\ i \in \{p, e\}, \tag{1}$$

where subscripts denote derivatives. Just as with any inverse demand function, B_i decreases with the number of users of type Y^i. This means that the marginal valuation of making the trip decreases with both formal and informal demand; that is:

$$B_{ij} = \frac{\partial^2 B(Y^p, Y^e)}{\partial Y^i \partial Y^j} < 0,\ i, j \in \{p, e\}. \tag{2}$$

In equilibrium, the marginal willingness to pay B_p equals the so-called generalized price, which is the total cost incurred by a representative formal user (see, Small & Verhoef, 2007, pp 83-85).

Therefore, the generalized price is computed as the fare τ plus the generalized cost $GC(Y, f)$, a function that values the waiting time, in-vehicle time, and crowding costs in monetary units:

$$B_p = \tau + GC(Y, f). \tag{3}$$

Following the public transport literature, we let $GC(Y, f)$ be an increasing function of the total demand $Y = Y^p + Y^e$ because it raises travel times and crowding, and a decreasing function of the frequency because it reduces waiting times (i.e., $GC_Y > 0$ and $GC_f < 0$; where, again, subscripts denote first derivatives).

Similarly, B_e equals the generalized price perceived by evaders. Of course, evaders do not incur any monetary cost; however, they may be inspected and fined with a certain probability, so they make decisions based on an Expected Value of the Fine (EVF), the generalized cost, and the personal evasion cost $EC(Y, f)$:

$$B_e = \pi + GC(Y, f) + EC(Y, f). \tag{4}$$

Assuming risk-neutral users with perfect information about the probability of being inspected, the EVF is computed as the product of the inspection rate and an exogenous fine. We could assume that the fine's value is endogenous; however, it would still be restricted by a maximum value given by law. It is straightforward to prove that this restriction is always binding because increasing the fine is costless (Buehler et al., 2017). The intuition is simple: one can always increase the value of the fine to deter fare evasion and save money on inspection, so the optimal fine always equals its upper bound. Furthermore, without loss of generality, we normalize the fine's value to one, so the EVF is given by π.

We also assume there is a personal evasion cost $EC(Y, f)$, which is a function that captures how service quality affects evading behavior in monetary units. Fare evaders usually excuse their behavior on the low service quality of public transport. Furthermore, crowding at bus stops, doors, and inside vehicles increase the anonymity perception and the so-called contagion effect, which makes dodging payment easier. The personal evasion cost $EC(Y, f)$ captures these effects; hence, it increases with frequency and decreases with the number of users (i.e., $EC_f > 0$, $EC_Y < 0$).

2.2. Social welfare

We assume a public operator that provides the necessary fleet for a given demand and frequency at a cost $OC(Y, f)$, an increasing function of the frequency and the total number of users (i.e., $OC_f > 0$, $OC_Y > 0$). The public operator collects the fare and receives a subsidy from the government if the revenues do not cover the operating cost. We also assume another public agency that performs the inspection activities at a cost $IC(\pi)$, an increasing function of the inspection rate.

We compute social welfare as the weighted sum of the consumers' surplus and the financial results of the public transport operator $\Pi^p = Y^p \cdot \tau - OC$ and the inspection agency $\Pi^e = Y^e \cdot \pi - IC$; that is:

$$W = CS + [1 + \lambda] \cdot [\Pi^p + \Pi^e], \tag{5}$$

where the consumers' surplus is given by the difference between the benefit function and the total cost perceived by formal and informal users

$$CS = B(Y^p, Y^e) - Y^p \cdot [\tau + GC(Y, f)] - Y^e \cdot [\pi + GC(Y, f) + EC(Y, f)]. \tag{6}$$

The parameter $\lambda \geq 0$ measures the tax distortion or the so-called shadow cost of public funds. This monetary distortion is due to the collection of local taxes on income, capital, or consumption (Laffont & Tirole, 1993). Therefore, if the government spends \$1 subsidizing the bus service, society pays $\$[1 + \lambda]$. Conversely, if the system's profit increases by \$1, the society earns $\$[1 + \lambda]$. We use this straightforward formulation rather than modeling general and labor equilibrium effects because it is simpler and allows for comparing our results more directly with previous literature that also adopts this approach (see, e.g., Proost & van Dender, 2008; Basso & Silva, 2014; Börjesson et al., 2019; Hörcher et al., 2020).

Replacing equilibrium conditions Eqs. (3) and (4), in the social welfare function Eq. (5), we can write the planner's welfare maximization problem as:

$$\begin{aligned} \underset{Y^p, Y^e, f}{\text{Max}} \quad & B(Y^p, Y^e) + \lambda \left[\sum_{i \in \{p,e\}} Y^i \cdot B_i \right] \\ & - [1 + \lambda] \Big[Y \cdot GC(Y, f) + Y^e \cdot EC(Y, f) + OC(Y, f) + IC(\pi(Y^e, Y^p, f)) \Big], \end{aligned} \tag{7}$$

where:

$$\pi(Y^e, Y^p, f) = B_e - [GC(Y, f) + EC(Y, f)]. \tag{8}$$

3. ANALYTICAL SOLUTION

This section describes and interprets the first-order conditions of the planner's maximization problem to understand how fare evasion distorts the standard public transport optimal provision. We assume that social welfare is strictly concave, so the first-order conditions characterize a unique interior solution for the optimal fare, inspection rate, and frequency. First, we analyze the welfare-maximizing fare and inspection rate when public funds are not costly, which is the most common assumption in the public transport literature. Second, we study how the cost of public funds impacts the optimal pricing and inspection rules. Third, we interpret the optimal frequency rule, and finally, we analyze how fare evasion affects optimal public transport subsidization.

3.1. Optimal price and inspection rate without tax distortions.

The welfare-maximizing fare when public funds are not costly or when the pricing of public transport does not need to be adjusted for other tax distortions (i.e., $\lambda = 0$), is given by:

$$\tau^* = [OC_Y + Y^p \cdot GC_Y] + Y^e [GC_Y + EC_Y] + IC_p. \tag{9}$$

where IC_p is the inspection cost savings due to a marginal increase in formal demand; that is:

$$IC_p = IC_\pi \frac{\partial \pi(Y^e, Y^p, f)}{\partial Y^p} = IC_\pi \left[B_{ep} - GC_Y - EC_Y\right] \leq 0. \tag{10}$$

Note that Eq. (9) is not a closed-form expression because the right-hand side depends on the demand and frequency. However, we can interpret the components of τ^* conditional on these variables.

The first term in square brackets is the sum of the marginal operating cost and the marginal external cost imposed on formal users. It is the welfare-maximizing public transport fare in a world without evasion; therefore, the sign of the remaining terms of Eq. (9) will indicate how fare evasion distorts the standard marginal-cost pricing rule.

The second term on the right-hand side of Eq. (9) is the marginal external cost that one additional user imposes on evaders. It has two components: the increase in the generalized cost due to the rise in crowding and travel times, and the reduction in the personal evasion cost because crowding at stops and buses raises the anonymity perception and makes evading easier. The empirical evidence suggests that this term is negative because increasing the total demand increases fare evasion (Salis et al., 2018; Currie & Delbosc, 2017; Porath & Galilea, 2020; Guarda, Galilea, Paget-Seekins, & Ortúzar, 2016). For example, in Santiago, a 1.0% increase in boardings and bus occupancy raises the number of evaders by 1.1% and 0.8%, respectively (Guarda, Galilea, Handy, et al., 2016). This indicates that one additional user reduces the cost perceived by evaders. Hence, the social planner discounts this effect to the optimal fare.

Finally, the third term is the inspection costs savings due to a marginal increase in formal demand. Under the standard assumption that the informal demand increases with public transport prices (i.e., $B_{ep} - GC_Y - EC_Y < 0$), the government saves money on inspection by reducing the fare. Therefore, these savings are discounted from the public transport optimal price. Note that the inspection savings increase with the marginal inspection cost; hence, the more costly the inspection is, the lower the optimal fare. Moreover, the greater the degree of substitution between paying and evading, the more significant the inspection savings are when the price is reduced. Thus, τ^* decreases with the substitutability between formal and informal demand.

Then, we can summarize the effect of fare evasion on the public transport optimal price when public funds are not costly, as follows: *If the informal demand for public transport increases with its price and total number of users, fare evasion distorts the optimal public transport pricing downwards.*

Eq. (9) also reveals that the welfare-maximizing inspection rate π^* plays a vital role in the design and pricing of public transport. Recalling that the Expected Value of the Fine (EVF) equals π, we compute π^* from the first-order conditions as:

$$\pi^* = \tau^* - IC_\pi \left[1 - \frac{B_{ep}}{B_{ee}}\right] \left[-B_{ee}\right]. \tag{11}$$

The relevant difference between τ^* and π^* is their impact on the inspection costs: reducing π impacts the inspection costs directly, an effect captured by B_{ee}. On the other hand, the effect of reducing τ depends on the degree of substitution between paying and evading, which is captured by B_{ep}. To illustrate this, let us analyze some particular cases. If paying and evading are perfect substitutes, $B_{ee} = B_{ep} = d'$, and $\tau^* = \pi^*$ (where d' is the derivative of the inverse demand function for the total demand). This occurs because reducing the fare has the same effect on the inspection costs as increasing the inspection rate. Furthermore, note from Eqs (9) and (11) that a perfectly inelastic total demand (i.e., $d' \to -\infty$) leads to the corner solution $\tau^* = \pi^* = 0$ because, in this case, the optimal price is the one that minimizes the inspection costs. If paying and evading are imperfect substitutes $B_{ep}/B_{ee} < 1$; thus, $\tau^* > \pi^*$. Moreover, the lower the degree of substitution between paying and evading, the higher the difference between τ^* and π^*, reaching its maximum value when paying and evading are independent (i.e., $B_{ep} = 0$).

3.2. Optimal price and inspection rate with costly public funds

The results in the previous section rely on the assumption that public funds do not have any cost to society. However, public funds have a non-negligible cost due to the distortion in the markets where the government collects taxes. Thus, this section analyzes how the shadow cost of public funds impacts the welfare-maximizing pricing rule.

The optimal fare and inspection rate when public funds are costly (i.e., $\lambda > 0$), denoted by ($\sim$), are given by the following Ramsey formula for interdependent demand:

$$\xi_p \left[\frac{\tilde{\tau} - \tau^*}{\tilde{\tau}} \right] = \xi_e \left[\frac{\tilde{\pi} - \pi^*}{\tilde{\pi}} \right] = \frac{\lambda}{1 + \lambda}. \tag{12}$$

Where τ^* and π^* are given by Eqs (9) and (11) but evaluated at the optimal demand and frequency, while ξ_p and ξ_e are the formal and informal demand *superelasticities*.[1]

Note that this result is similar to the one obtained by Hörcher et al. (2020) when studying the effect of agglomeration economies in the public transport provision with a substitute mode. The key difference is that instead of two modes, we have two different types of consumption: formal and informal. However, the interpretation is similar: the optimal price and inspection rate are set above τ^* and π^* because revenues reduce the need for costly taxes. Moreover, the pricing and inspection rules increase with λ and depend on both the direct and cross elasticities because increasing the public transport price increases the operator's revenues but also raises revenues from fines, an effect captured by the demand *superelasticity*.

Eq. (12) shows a trade-off between the benefit of reducing the public transport price, captured by τ^*, and the cost to society in terms of distortionary taxes. Therefore, the value of the optimal fare depends on which of these two effects dominates. We can understand this trade-off by analyzing

[1] The demand *superelasticity*, first derived by (Boiteux, 1956), is usually defined in terms of the direct and cross demand elasticities as:

$$\xi_i = \frac{\eta_{ii}\eta_{jj} - \eta_{ij}\eta_{ji}}{\eta_{ji} - \eta_{jj}} > 0,$$

where $\eta_{ii} = \dfrac{\partial Y^i}{\partial p^i}\dfrac{p^i}{Y^i}$ and $\eta_{ij} = \dfrac{\partial Y^i}{\partial p^j}\dfrac{p^j}{Y^i}$ (Laffont & Tirole, 1993).

the particular cases when paying and evading are perfect substitutes, which results in the following pricing rule:

$$\tilde{\tau} = \tilde{\pi} = \tau^* + \frac{\lambda}{1+\lambda}\left[-d' \cdot Y\right]. \tag{13}$$

Where the last term on the right-hand side of Eq. (13) is the additional charge due to the shadow cost of public funds. Note from Eqs. (9) and (13) that an increase in the slope of the total demand has two opposite effects: On the one hand, it reduces τ^* because it raises the inspection cost savings, but on the other, it increases the charge due to costly public funds. Furthermore, when the total demand is perfectly inelastic, we get $\tilde{\tau} = \tilde{\pi} = 0$ if the former effect dominates and $\tilde{\tau} = \tilde{\pi} = 1$ (i.e., the maximum EVF) if the cost of public funds does it. This illustrates that, under certain conditions, fare evasion can create a strong incentive to reduce public transport prices that may overcome the cost of public funds. However, when the cost of public funds dominates, the optimal fare is set above the standard marginal-cost pricing rule to reduce the need for costly taxes.

3.3. Optimal frequency

From the first-order conditions, we get the optimal frequency rule given by Eq. (14), where $\tilde{Y}^i$ denotes the optimal number of formal or informal users:

$$OC_f = -\tilde{Y}^P GC_f + \Pi^e_f. \tag{14}$$

The social planner increases the frequency until the rise in the operating costs equals the reduction in the formal users' cost plus the change in the profit of the inspection agency, which is given by:

$$\Pi^e_f = -[\tilde{Y}^e - IC_\pi][GC_f + EC_f]. \tag{15}$$

Where the term in the first square brackets of Eq. (15) is the difference between the marginal revenues from fines $\tilde{Y}^e$ (recalling that the fine equals one), and the marginal inspection cost IC_π; while the term in the second square brackets is the change in the costs perceived by evaders due to a marginal increase in the frequency.

Note that optimal frequency is obtained holding $\tilde{Y}^i$ constant. This is correct because the optimal pricing and inspection rules, given by Eq. (12), ensure that any marginal benefits or costs of increasing the frequency via changes in the number of users have zero effect on social welfare (Small & Verhoef, 2007, p. 164). In other words, any variation in the operating and users' costs due to a demand change is captured by the social planner through the optimal fare and inspection rate. This has two consequences: first, the shadow cost of public funds affects the optimal frequency rule via changes in the optimal price and inspection rate (Hörcher et al., 2020; Zhang et al., 2020; Sun et al., 2016); second, the effect of fare evasion on the optimal frequency is captured by Π^e_f.

To illustrate the latter, let us assume for a moment that the cost perceived by evaders does not change with quality (i.e., $GC_f + EC_f = 0$); therefore, increasing the frequency does not change the inspection profit (i.e., $\Pi^e_f = 0$). In this case, Eq. (14) becomes the standard frequency rule $OC_f = -Y^p \cdot GC_f$. Hence, Π^e_f captures the distortion in the optimal frequency rule that emerges as a consequence of fare evasion. If it is positive, fare evasion sums an additional benefit to the usual public transport frequency rule that induces higher service quality; if it is negative, fare evasion has the opposite effect.

The fare evasion literature suggests that improving the frequency increases the evader's cost and, therefore, reduces fare evasion (Busco et al., 2021; Barabino et al., 2015; Guzman et al., 2021). In this case, Π^e_f is positive if reducing the inspection rate increases the inspection profit. Thus, we can summarize the impact of fare evasion in the optimal frequency as follows: *If improving the frequency of public transport reduces informal demand, fare evasion increases the optimal public transport frequency as long as the marginal inspection cost exceeds the marginal revenue of fines.*

The intuition behind this result is the following: if increasing the frequency makes evasion less attractive, the social planner captures this effect through a reduction in the inspection rate, which in turn, increases social welfare if the fall in the inspection cost IC_π is larger than the fall in revenues from fines, $\tilde{Y}^e$. In other words, if fare evaders are worst off when the frequency increases, the social planner uses service quality to control fare evasion and save money on inspection. This suggests that transport authorities may have strong incentives to improve service quality when the inspection is too costly or ineffective or when revenues from fines are negligible.

3.4. Revisiting the Mohring effect and public transport subsidies

An important result in the public transport literature is the so-called Mohring effect (1972; 1976). It states that, under certain technical conditions, the sum of operating and users' costs grows less than proportional with demand, leading to an optimal fare that does not cover operating costs and requires subsidies (H. E. Silva, 2021). The source of the Mohring effect is that the optimal frequency increases with demand; however, since previous studies do not consider fare evasion, it is relevant to analyze if this result holds in our model. To do this, let us treat quantities Y^p and Y^e as parameters. Then, totally differentiating the optimal frequency rule, we get:

$$\frac{df}{dY^p} = -\frac{W_{fp}}{W_{ff}} = -\frac{\Pi^e_{fp} - \{GC_f + Y^p \cdot GC_{fY} + OC_{fY}\}}{\Pi^e_{ff} - \{Y^p \cdot GC_{ff} + OC_{ff}\}}. \tag{16}$$

The denominator on the right-hand side of Eq. (16) is negative to satisfy the second-order conditions. Therefore, the sign of df/dY^p is given by the sign of the welfare's cross-derivative W_{fp}, which is positive if the following condition holds:

$$\Pi^e_{fp} > GC_f + Y^p GC_{fY} + OC_{fY}. \tag{17}$$

Thus, whether the Mohring effect is active or not depends on the functional forms of the cost functions, specifically, on the second derivatives of generalized, evasion, inspection, and operating costs.

Now, let us replace the optimal price in the financial result of the public transport operator to get the optimal subsidy per formal user:

$$S = \underbrace{OC/Y^p - [OC_Y + Y^p \cdot GC_Y]}_{\text{Mohring Effect (+)}} + \underbrace{\{-IC_p - Y^e[GC_Y + EC_Y]\}}_{\text{Fare evasion (+)}} + \underbrace{\frac{\lambda}{1+\lambda}\left[B_{pp} \cdot Y^p + B_{ep} \cdot Y^e\right]}_{\text{Cost of public funds (-)}}. \tag{18}$$

The first component of the optimal subsidy is the difference between the average and marginal costs. It is the optimal subsidy when fare evasion and costly public funds are not considered in the analysis (Jara-Díaz & Gschwender, 2005); thus, it is positive if the Mohring effect is active. The second component is the marginal benefit of reducing the fare due to evasion, which is positive and has two terms: the inspection cost savings and the fall in the evaders' total costs. Finally, the third component is negative since it is the cost of subsidizing public transport in terms of distortionary taxes. When $\lambda = 0$, this distortion equals zero because subsidies are just transfers between the government and the operator; however, if $\lambda > 0$, it reduces the optimal subsidy.

Then, we have two forces that put upward pressure on public transport subsidization: the Mohring effect and the benefits of reducing fare evasion. On the other hand, the cost of public funds pushes in the opposite direction. Therefore, the subsidy amount, if needed, will depend on which of these effects dominates.

4. NUMERICAL ANALYSIS

In the previous section, we found analytical expressions that characterize the optimal price, inspection rate, and frequency of public transport. However, since we do not define functional forms for the demand and cost functions, we can not find closed-form formulas for the decision variables to quantify how fare evasion impacts the optimal public transport provision. Thus, in this section, we perform numerical simulations aiming to reflect a representative bus system with a high evasion rate. To do this, we represent the commuters' choice using a Nested-Logit (NL), assume the generalized and operating cost functions proposed by Jara-Díaz & Gschwender (2003), a personal evasion cost with constant elasticity with respect to boardings and bus occupancy, and a linear inspection cost function.

Fig. 1 summarizes the results of the numerical simulations. It shows the resulting market share and evasion rate. The baseline scenario is marked by point B and the optimal solution is marked by Point O. The color lines show the sensitivity with respect to: (i) the operating cost multiplier γ, (ii) SCPF λ, (iii) the marginal inspection cost ω, and (iv) the evasion cost elasticity with respect to boardings β_1.

Note that the market share in the optimal solution is only two percentage points lower than in the baseline scenario; however, the evasion rate falls from 30% to less than 20%. This means that the total number of users in the baseline scenario is close to optimal; however, the proportion of these trips that do not pay the fare is ten percentage points above the welfare-maximizing level. Thus, the optimal inspection rate is five times higher than in the baseline scenario, which is in line with previous fare evasion literature (Barabino & Salis, 2019). The main consequence of this reduction

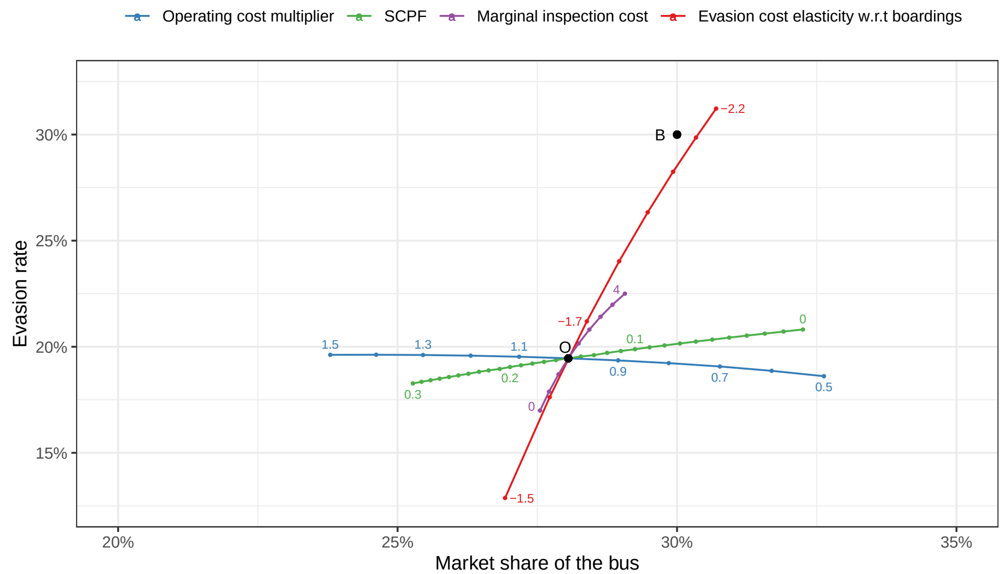

Figure 1: Numerical results

in the evasion rate is that revenues from fares increase by 20%, so the cost recovery ratio rises from 60% in the baseline scenario to 75% in the optimal solution. This means that the public transport subsidization in the baseline scenario is above the optimal level due to the revenue losses from fare evasion.

Unlike previous literature that considers the cost of public funds (see, e.g., Basso & Silva, 2014), the optimal fare does not cover the operating cost in all our simulations. This occurs for two reasons: first, straightforward computations show that the Mohring effect is active, and second, increasing subsidies reduces inspection and evaders' costs. As shown in Eq. (18), these two forces create a strong incentive to raise the operational subsidy that, in our simulations, dominates the cost of public funds.

5. CONCLUSION

In this paper, we have developed what we believe is the first theoretical model to study how fare evasion affects welfare-maximizing public transport price and quality. The main finding of our analysis is that reducing fare evasion may be a solid new argument for public transport subsidization. Numerical simulations confirm that the incentive to reduce fare evasion and the Mohring effect can justify subsidies considering realistic values of the shadow cost of public funds. The policy implication of this result is that a significant reduction in the public transport price may be an efficient strategy to deal with fare evasion in cities where other measures, such as inspection, awareness campaigns, and deterrence, have proven ineffective or too costly. Moreover, our results contribute to the ongoing policy discussion about public transport subsidization in the developing world, where fare evasion is more relevant for transport authorities and scholars.

In our model, evaders heterogeneity is captured on the benefit function, which allows us to obtain simple analytical expressions for the optimal design and pricing. We acknowledge, though, that evaders' heterogeneity is more complex than this simple representation. Therefore, a natural extension to our work is to consider evaders' heterogeneity regarding intentionality, income, and other socio-economic characteristics relevant to the analysis. Spatial variations of the demand conditions may also affect our results since the average crowding experienced by users is higher than the average bus occupancy. This occurs because travelers are disproportionally concentrated in more crowded vehicles, so fare evasion would also be concentrated where crowding is higher. Thus, including a spatial dimension in the analysis could be an interesting extension of our work.

Another possible extension is to consider the automobile as a substitute mode since unpriced car externalities significantly impact public transport design and pricing. Furthermore, a more complex but realistic approach, aiming to represent a specific city, should be considered to study the interaction between fare evasion and other modes, markets, and policies. We also see the study of non-linear pricing, such as travel passes, as a possible extension since the marginal price paid by pass owners is zero.

Our results also rely on the assumption that a public operator provides the bus service. However, public transport is usually operated by private firms whose incomes may be affected by fare evasion. Therefore, regulating private operators can also be an interesting extension of our model. Finally, we acknowledge that fare evasion depends on social, political, and cultural factors beyond the scope of this paper. Although challenging, incorporating these dimensions into fare evasion modeling may be an interesting avenue for future research.

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