A MARKOVIAN APPROACH FOR DYNAMIC TRANSIT ASSIGNMENT: RESULTS IN A SUBNETWORK OF SANTIAGO

Ricardo de la Paz Guala, Universidad de Chile – ricardo.delapaz@uchile.cl
Marcela Munizaga, Universidad de Chile – mamuniza@ing.uchile.cl
Jaqueline Arriagada, Universidad de Chile, (currently at University of Leeds) – j.arriagada@leeds.ac.uk

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ABSTRACT

We develop the Markovian dynamic transit assignment (MDTrA) modelling framework for large multi-modal networks to incorporate into the transit assignment analysis the dynamic aspects that come from the time-dependent relationship between demand and supply and the stochasticity that comes from differences in passenger's costs perception, measurement errors, and other sources of uncertainty. The intuition is that passengers choose their routes by a recursive arc-choice process, according to the expected minimum costs from their current node to their destinations. Our approach presents an important oportunity to use smartcard data, as demand profiles and users's choices over time are obtained from it.

1. INTRODUCTION

Transit assignment models play a fundamental role in the planning of public transport systems. They allow to estimate the flow of passengers for each alternative route to reach a certain destination from a given origin. Developing a transit assignment model is a complex task since it involves emulating the passengers' route decision making process and identifying the attributes of the different routes. This challenge is even bigger in a dense multimodal transit network, which usually contains hundreds of routes. Currently, the available data from different types of sources (such as smartcards), the inherent uncertainty in users' choices, and the natural time-dependence of demand and supply of transit transport networks, pushes researchers to deal with dynamic and stochastic dimensions to contribute with approaches able to interpretate our current transport context.

Transit assignment studies are mostly based on *deterministic passenger equilibrium* (*UE*) (Nguyen & Pallottino, 1988; Spiess & Florian, 1989; de Cea & Fernández, 1993; Cominetti & Correa, 2001; Cepeda et al., 2006), assuming passengers with perfect knowledge about route costs. Therefore, algorithms find the shortest hyperpath Nguyen & Pallottino (1988) and all demand is assigned among these routes, with passengers taking the first line that arrives at the stop (Spiess & Florian, 1989; Nguyen & Pallottino, 1988). Literature based on *stochastic passenger equilibrium* (*SUE*) (Lam et al., 1999; Kiencke & Nielsen, 2000; Nielsen & Frederiksen, 2006; Yang & Lam, 2006) assumes imperfect information about route costs and, therefore, algorithms use *random utility maximization* (*RUM*) for assigning demand, where passengers choose routes with higher perceived utility.

In assignment literature we find the Markovian approach to address uncertainty in users' choices. Its intuition is that users construct their routes by making recursive arc choices, rather than choose a fixed route. It is introduced by Baillon & Cominetti (2008) with the *Markovian traffic equilibrium* (*MTE*) for the static case in private transport networks. The concept is later applied to generate the first stochastic transit assigment model with a Markovian approach (*STE*) in Cortés et al. (2013), later extended in Pineda et al. (2016) by adding private transportation (*STP*). Later on, in de la Paz Guala (2020), the Markovian approach is adapted for private transport networks with time-dependent demand. In all these contributions the arc-choice model considers as the users' choice criterion the expected minimum costs of the remaining trip, generating nested cost operators that are able to avoid path enumeration and allow working with overlapping routes.

When it comes to the use of real data, model calibration and validation process is fundamental to set the model's parameters and to evaluate if it is generating passenger flows according to real passengers' behaviour. Although emerging technologies, such as *automatic fare collection* (*AFC*) and *automatic vehicle location* (*AVL*) systems, offer an opportunity to deal with these problems, there are relatively few studies for calibration and validation purposes. In this line, Tavassoli et al. (2020) proposed a framework to calibrate and validate existing transit assignment models using smartcard transaction data and AVL data from public transport systems.

Motivated by the contributions in transit assignment models, the referred Markovian approaches, and the use of smartcard data, the main goal of this paper is to propose a Markovian modelling framework to address the dynamic aspects from time-dependent demand and supply, and the uncertainty from users's choices, where smartcard data is used to estimate fundamental information for the model. To accomplish this, we present: the *Markovian dynamic transit assignment* (*MDTrA*) model, an illustrative example of how it works, an example of how to use real smartcard data to estimate dynamic demand profiles, and a solution method for the proposed model.

2. MARKOVIAN DYNAMIC TRANSIT ASSIGNMENT MODEL

The main result is the Markovian dynamic transit assignment (MDTrA) model, an integration of the MDTA modelling framework by de la Paz Guala (2020), a dynamic adaptation of the MTE (Baillon & Cominetti, 2008), both for private transportation, and the approach in Cortés et al. (2013) and Pineda et al. (2016), both for public transportation. The intuition is that, given exogonous time-dependent demand and supply profiles, passengers, at each stop/station they are currently at, will choose the next arc to move forward considering the expected minimum costs from the remaining trip, following a logit rule and wether the arc is reasonable or not (subsection 2.2).

The MDTrA model has a *demand and supply profiles*, a *cost and time model*, and a *arc-choice model*. Before developing each part, we first address the digraph that our formulation is based on, and then we introduce the *reasonability* concept, a defining feature of our work.

2.1. Transit network's underlying digraph

To formulate the MDTrA model, we first generate a digraph from the transport system's underlying service network. For that, we consider: the set of all stops/stations (stops from now on, for simplicity) $\mathcal{N}$; the set of lines of the system, $\mathcal{L}$; the sequence of stops that each line $l \in \mathcal{L}$ serves, $\mathcal{N}_l \subseteq \mathcal{N}$, $\forall l \in \mathcal{L}$; the set of lines that serve each stopt $s \in \mathcal{N}$, $\mathcal{L}_s \subseteq \mathcal{L}$, $\forall s \in \mathcal{N}$.

First, we define what we consider as a node. As shown later in section 5, the application of our approach features the use of smartcard data and, because of the way that information is obtained, origins and destinations are understood as stops in the public transport system. Previous to the modelling, we process the spatial information of the network to reduce the amount of nodes that will be considered, respecting logical considerations. We do so by merging stops that are closer to each other within a certain ratio considered to be narrow enough to make the merging rational, in the sense that to enter/exit through any of the stops containted within that ratio is no significantly different to enter/exit through a different stop within the same area. Now, following that intuiton and consering the demand that is associated to each stop, the process is briefly described as follows:

1. For all stops, visited one by one, check if within a given ratio there are more stops;
2. If there are, merge them to generate an aggregated node that, at the end, will represent all stops within that area that are served by different services;
3. The agreggated node is now served by the union of services associated to each merged stop;
4. The aggregated demand of all merged stops is the demand of the new aggregated stop.

In Appendix A.1 we present a pseudocode for this routine, where the scenario in which a stop that hasn't been merged yet lies within the ratio of multiple other stops is properly addressed.

Let us denote the set of merged stops as $\mathcal{N}^M$, the set of merged lines as $\mathcal{L}^M$, the set of merged lines $i \in \mathcal{N}^M$ as $\mathcal{L}_i^M$, and the demand functions of each pair of merged stops (o, d) as $\mathcal{D}_{o,d}^M(\cdot)$. Considering this, we construct a digraph $G = (N, A)$ by applying the following criteria:

- **Set of nodes**: N is the union of:
  - the set N_o, denoted as *stop nodes*: For each merged stop $i \in N^M$ a node $i \in N_O$ represents it (for simplicity, we keep the same notation;
  - the set N_r, denoted as *replicated service nodes*: For each node $i \in N_o$ representing a stop $i \in \mathcal{N}^M$ and for each line $l \in L_i^M$, a node i_l is added, representing that line l serves from/to i;
- **Set of arcs**: A is the union of:
  - the set of *boarding arcs* A_b, that contains, for each $i \in N_o$ and for each replicated service nodes i_l, the arc (i, i_l) that represents boarding line l at i;

- the set of *alighting arcs* A_a, that contains, for each node $i \in N_o$ and for each one of its i_l, the arc (i_l, i) that represents alighting from line l at i;
- the set of *in-vehicle arcs* A_v, that contains, for each line $l \in \mathcal{L}^M$, the arc (i_k, i_{k+1}), where i_k and i_{k+1} are nodes associated with consetutive merged stops in the sequence of stops served by line l, that represents moving from i_k to i_{k+1} using line l;
- the set of *walking arcs*, A_w, that contains the arc (i, j) if merged stop j is considered to be walkable from merged stop i.

As an example, consider a network that, after merging, has: stops $1, 2, 3$ and 4; lines l_1, l_2 and l_3; serving sequences of nodes $\{1, 3, 4\}$, $\{1, 2, 3\}$ and $\{1, 2, 4\}$, respectively; and stop 4 is walkable from stop 3. Then, this transport system has as underlying digraph the one shown in Figure 1.

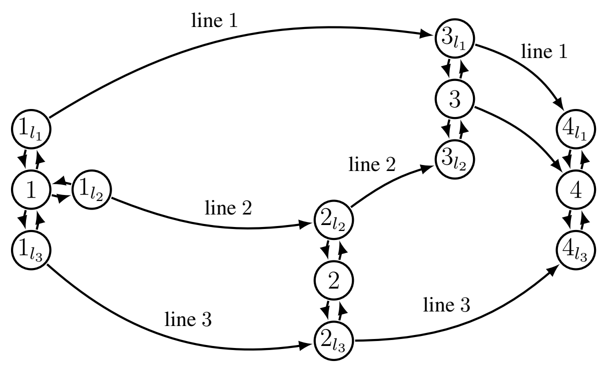

Figure 1: Transport network's underlying digraph G

We emphasize that the base of the formulation is the digraph $G = (N, A)$, thus, several previously used notations (defintions with cursive notations, for example), will not be present in the model. The term stop can be used to refer to its corresponding node in the underlying digraph.

2.2. Reasonable arcs towards destinations

When passengers have to choose between options, by default, each one has a positive probability of being chosen. In reality, this does not generally hold, as some options may not be eligible for passengers, depending on their criteria. Addressing this, Dial (1971) defined, for static stochastic traffic assignment in private transport networks, that given an O-D pair (o, d), a route from node i to node j is a reasonable route for (o, d) if the minimum cost of going from o to i is less than the minimum cost of going from o to j and, simultaneously, the minimum cost of going from j to d is less than the minimum cost of going from i to d. In de la Paz Guala (2020) this concept is adapted, defining it over arcs (instead of routes) and according to destinations (instead of O-D pairs), stating that, given a destination node d, an arc (i, j) is a reasonable arc towards d if the minimum cost of going from node j to d is less or equal to the minimum cost of going from node i to d. On other line of work, Nuzzolo & Comi (2017) introduce the *master hyperpaths*, where, among all the strategies, the master hyperpath is formed by those that are actually considered by passengers.

In this paper, we introduce a *reasonability* concept, as in de la Paz Guala (2020), to reduce the route options to those that are actually considered by passengers. Given a destination node d, a line service l, and two consecutive stops i and j (served by l), we say that (i_l, j_l) is a *reasonable arc towards* d if the minimum cost of going from j to d is less or equal to the minimum cost of going from i to d. Also, the boarding and alighting arcs associated to a reasonable arc, $(i, i_l), (i_l, i), (j, j_l), (j_l, j)$, are also reasonable (and vice versa). The set of arcs reasonable towards d is denoted as R_d.

2.3. Building the MDTrA model

Now, we define the parts of the model considering an analyzed period of time $[0, T]$.

2.3.1 Demand and supply profiles

Consider a set of O-D pairs OD. The demand profile is represented by the exgoneous time-dependent functions $\mathcal{D}_{(o,d)}(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$, for all $(o, d) \in OD$. Given $t \geq 0$, $\mathcal{D}_{(o,d)}(t)$ represents the demand that, at time t, is generated at o and goes to d.

The supply profile is the exgoneous time-dependent mean frecquency of lines that serve the transit network. Given a replicated service node $i_l \in N_r$ and its underlying line l, the function $\phi_{i_l}(\cdot) : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ represents the mean frequency of l at i. Note that this function will be equal for all nodes served by the same line, this is, for all i served by l $\phi_{i_l}(\cdot) = \phi_l(\cdot)$.

2.3.2 Time and cost models

Time functions are fundamental, as they locate passengers in time, and are a component of the cost functions used to compute the expected minimum costs. It is worth remarking that, given our arc-based approach, times and costs are defined for each arc and, thus, we do not require them to be defined for routes. This allows us to work with overlapping routes, not needing the condition of route's times and costs independence, as it is widely needed in assignment literature.

For each arc $a \in A$, depending on its type, we define the time function $T_a(t)$ and, according to each destination node $d \in D$, a cost function $C_{ad}(t)$, $t \in [0, T]$. Both functions are heavily related as, at each arc, part of its cost is defined by its underlying times. Then, for $t \in [0, T]$, we have:

- For boarding arcs $a = (i, i_l) \in A_b$, $T_a(t)$ is given by the addition of:
  - the time a passenger takes to access stop i and reach a representative physical point where line l arrives, b_a;
  - the average time that the passengers waits for line l, given by $\frac{1}{\phi_l(b_a+t)}$.

  Now, independently of the destination $d \in D$, $C_{ad}(t)$ is the direct transformation of times into costs (using the factors α and β, respectively).

- For alighting arcs $a = (i_s, i) \in A_a$, $T_a(t)$ is given by the time that takes to alight line l and exit stop i, q_a. Now, given a destination node d, $C_{ad}(t)$ has two cases:
  1) if the alighting happens at destination node d, then the cost is given by the direct transformation of time into cost by using factor δ;
  2) otherwise, if the alighting happens at node different than d, then a penalty p_a is added to the transformation of time into cost.
- For in-vehicle arcs $a = (i_s, j_s) \in A_v$, $T_a(t)$ is given by the time that takes to travel from i to j using line l, t_a. Now, given a destination node $d \in D$, $C_{ad}(t)$ has two components:
  - the direct transformation of t_a into cost (using the factor γ);
  - a cost associated to overcrowding, that depends proportionally (factor H_a) on the total inflow assigned to arc a at instant t, $\sum_{d' \in D} E_a^{d'}(t)$ (addressed in subsecion 2.3.3).
- For walking arcs $a = (i, j) \in A_w$, $T_a(t)$ is the time w_a and, regardless the destination $d \in D$, $C_{ad}(t)$ is the time directly transformed into cost (using the factor ρ).

Then, for each arc $a = (i, j) \in A$ and at a time $t \in [0, T]$, the time that takes to travel through arc a having entered it at time t is given by:

$$T_a(t) = \begin{cases} b_a + \frac{1}{\phi_l(b_a+t)}, & \text{if } a = (i, i_l) \in A_b, \\ q_a, & \text{if } a = (i_1, i) \in A_a, \\ t_a, & \text{if } a = (i_l, j_l) \in A_v, \\ w_a, & \text{if } a = (i, j) \in A_w, \end{cases} \tag{1}$$

while, for each $d \in D$, the cost of travelling through arc a while heading to destination node d, having entered the arc at t, is given by:

$$C_{ad}(t) = \begin{cases} \alpha b_a + \frac{\beta}{\phi_l(b_a+t)}, & \text{if } a = (i, i_l) \in A_b, \\ \delta q_a, & \text{if } a = (i_l, i) \in A_a \text{ and } j = d, \\ \delta q_a + p_a, & \text{if } a = (i_l, i) \in A_a \text{ and } j \neq d, \\ \gamma t_a + H_a \displaystyle\sum_{d' \in D} E_a^{d'}(t), & \text{if } a = (i_l, j_l) \in A_v, \\ \rho w_a, & \text{if } a = (i, j) \in A_w. \end{cases} \tag{2}$$

2.3.3 The choice model

The arc-choice model, according to the expected minimum costs to each destination, assigns inflow from each node among its outgoing reasonable arcs. It has delicate considerations as, for example, not all arcs are necessary reasonable and, at a given time, if inflow rate has been assigned to an arc (i, j) it can not be immediatly assigned back through its eventual inverse arc (j, i) .

Given a destination d, an arc $a = (i, j)$, and a time t, the exptected minimum cost of using a to go to d, having entered at t, denoted as $Z_{ad}(t)$, is given by the cost of a going to d at t, $C_{ad}(t)$, plus the expected minimum cost from j to d (arriving at j at $t + T_a(t)$). The latter considers all the outgoing reasonable arcs to d from node j (except for (j, i)). Thus, the construction of the expected minimum costs is recursive and, even though route enumeration is not needed, its second term keeps the information of the exptected minimum cost of what is left of the trip to d.

We first efine some sets. Consider a node $i \in N$, a destination node $d \in D$, and the set of reasonable arcs towards d, R^d. Let us denote two subsets of arcs in R^d, the outgoing arcs from i and the incoming arcs to i, $R^{d}{}_i^+ = \left\{(i,j) \in A_i^+ : (i,j) \in R^d\right\}$ and $R^{d}{}_i^- = \left\{(j,i) \in A_i^- : (j,i) \in R^d\right\}$, respectively. Now, we denote the subsets $R^{d}{}_i^+$ and $R^{d}{}_i^-$ whose inverse is also reasonable towards d as $B^{d}{}_i^+ = \left\{(i,j) \in R^{d}{}_i^+ : (j,i) \in R^{d}{}_i^-\right\}$ and $B^{d}{}_i^- = \left\{(j,i) \in R^{d}{}_i^- : (i,j) \in R^{d}{}_i^+\right\}$, respectively.

Then, being θ the dispersion parameter, for each destination $d \in D$, for each arc $a = (i, j) \in A$, and at each time $t \in [0, T]$, the expected minimum cost of using a going to d, entering at t, is:

$$Z_{ad}(t) = \begin{cases} C_{ad}(t), & \text{if } j = d, \\ C_{ad}(t) - \frac{1}{\theta} \ln \left(\displaystyle\sum_{b \in R_j^d \setminus \{(j,i)\}} \exp\left(-\theta Z_{bd}\left(t + C_{ad}\left(t\right)\right)\right) \right), & \text{otherwise.} \end{cases} \tag{3}$$

Now, given a destination d, a node i and an instant t, the outflow rate of all incoming arcs b arriving to i, denoted as $G_{bd}(t)$ (equation 5), and the demand at i at time t, both with destination d, are aggregated and then assigned as inflow among the outgoing reasonable arcs from i.

Given a destination node $d \in D$, for each node $i \in N - d$, and at each instant $t \in [0, T]$, then, we have that, for all $a = (i, j) \in A_i^+$:

$$E_{ad}(t) = \begin{cases} \dfrac{e^{-\theta Z_{ad}(t)}}{\displaystyle\sum_{b \in R^{d}{}_i^+} e^{-\theta Z_{bd}(t)}} \left(\displaystyle\sum_{b \in A_i^- \setminus B^{d}{}_i^-} G_{bd}(t) + \mathcal{D}_{(i,d)}(t) \right) + \displaystyle\sum_{(i,m) \in B^{d}{}_i^+} \left(\dfrac{e^{-\theta Z_{ad}(t)}}{\displaystyle\sum_{b \in R^{d}{}_i^+ \setminus \{(i,m)\}} e^{-\theta Z_{bd}(t)}} G_{(m,i),d}(t) \right), \\ \dfrac{e^{-\theta Z_{ad}(t)}}{\displaystyle\sum_{b \in R^{d}{}_i^+} e^{-\theta Z_{bd}(t)}} \left(\displaystyle\sum_{b \in A_i^- \setminus B^{d}{}_i^-} G_{bd}(t) + \mathcal{D}_{(i,d)}(t) \right) + \displaystyle\sum_{(i,m) \in B^{d}{}_i^+ \setminus \{a\}} \left(\dfrac{e^{-\theta Z_{ad}(t)}}{\displaystyle\sum_{b \in R^{d}{}_i^+ \setminus \{(i,m)\}} e^{-\theta Z_{bd}(t)}} G_{(m,i),d}(t) \right), \\ 0. \end{cases} \tag{4}$$

if $a \in R^{d}{}_i^+ \setminus B_i^+$, or if $a \in B_i^+$, or otherwise, respectively.

Now, when inflow rate enters an arc $a = (i, j)$ through i, it reaches j and exits as outflow rate after the travel time of the arc, $T_a(t)$. Thus, given a destination $d \in D$, for each arc $a \in A$ and at each instant $t \in [0, T]$, the inflow and the outflow rates of arc a going to d are related as follows:

$$G_{ad}(t + T_a(t)) = E_{ad}(t). \tag{5}$$

3. ILLUSTRATIVE EXAMPLE

Consider digraph G from Figure 1, where metro line 1 serves node 1, 3 and 4; metro line 2 serves nodes 1, 2 and 3; and bus line 3 serves nodes 1, 2 and 4. Also, node 4 is walkable from node 3. A demand rate of passengers with origin at node 1 going to node 4 is shown in Figure 2. The suppply is defined by the constant mean frequencies of lines 1, 2 and 3, given by $\phi_1(t) = 1\ \frac{metro}{min}$, $\phi_2(t)$ =0.5 $\frac{metro}{min}$, and $\phi_3(t)$ =0.2 $\frac{bus}{min}$, respectively.

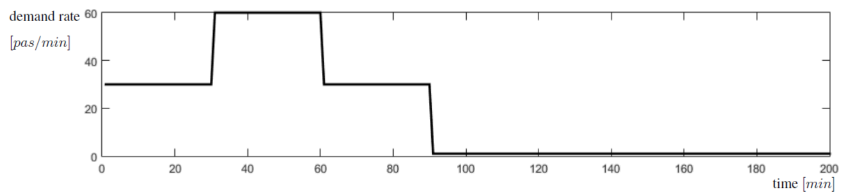

Figure 2: Demand rate from node 1 to node 4

In this example, costs of arcs are directly defined by times, according to equations 1 and 2. Thus, cost and time differ only when a is an alighting arc that does not arrive to node 4. Table 1 shows: access times; exit times; transfer penalties for not alighting at node 4; and mean waiting times of lines. Table 2 shows costs associated to in-vehicle and walking arcs.

	access	exit	transfer penalty	waiting
node 1 - line 1	3	3	5	1
node 1 - line 2	3	3	5	2
node 1 - line 3	1	1	10	5
node 2 - line 2	2	2	5	2
node 2 - line 3	1	1	10	5
node 3 - line 1	2	2	5	1
node 3 - line 2	2	2	5	2
node 4 - line 1	2	2	0	1
node 4 - line 3	1	1	0	5

Table 1: Costs $[min]$ of node-line interactions

	travel time
node 1 to 2 - line 2	18
node 1 to 2 - line 3	30
node 1 to 3 - line 1	20
node 2 to 3 - line 2	10
node 2 to 4 - line 3	25
node 3 to 4 - line 1	25
node 3 to 4 - walking	35

Table 2: Travel times $[min]$ between nodes

Then, the time and cost $[min]$ of using each arc (going to destination 4) are shown in Figure 3:

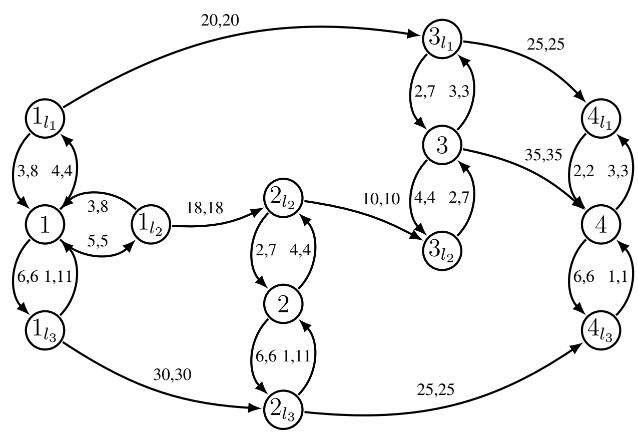

Figure 3: Travel times and costs $[min]$ of each arc

Then, the expecetd minimum costs to destination 4 are those presented in Table 3.

(i,j)	$Z_{(i,j),4}(t)$
$(1,1_{l_1})$	43.26
$(1,1_{l_2})$	46.30
$(1,1_{l_3})$	56.84
$(1_{l1},3_{l_1})$	39.26
$(1_{l_2},2_{l_2})$	41.30
$(1_{l_3},2_{l_3})$	50.84

(i,j)	$Z_{(i,j),4}(t)$
$(2,2_{l_2})$	39.84
$(2,2_{l_3})$	32
$(2_{l_2},2)$	39
$(2_{l_2},3_{l_2})$	35.48
$(2_{l_3},2)$	50.48
$(2_{l_3},4_{l_3})$	26

(i,j)	$Z_{(i,j),4}(t)$
$(3,4)$	35
$(3,3_{l_1})$	30
$(3_{l_1},3)$	42
$(3_{l_1},4_{l_1})$	27
$(3_{l_2},3)$	25.48
$(4_{l_1},4)$	2
$(4_{l_3},4)$	1

Table 3: Expected minimum costs $[min]$, from arcs and from nodes, to node 4.

Figure 4 depicts how the flow of passengers is assigned, at each node, among the different options to go to node 4, highlighting the dynamic nature of the problem. As flow rate originates dynamically at node 1, it travels through the arcs arriving at different times to other nodes. The evolution of the inflow rates from the nodess transforms the curve of the demand rate generated at node 1 (Figure 2) into the curve of flow rate arriving to node 4 (Figure 5).

4. MDTRA ALGORITHM FOR TRANSIT NETWORKS

We propose the *Markovian dynamic transit assignment* (*MDTrA*) algorithm, a dynamic programming method that solves a discrete version of the problem. It adapts the MDTA algorithm (de la Paz Guala, 2020), based in a repeated and reversed-step version of *Dial's algorithm* (Dial, 1971).

The MDTrA algorithm has as inputs: (1) the digraph (N, A); (2) the set of destinations, $D \subseteq N$; (3) the demand-supply profile $(\mathcal{D}(\cdot), \phi(\cdot))$; (4) an array P of access times, exit times and transfer penalties; (5) an array of the travel times between consecutive nodes for all lines, V; (6) the length of the time period, T; (7) the timestep size of the discretization, Δt; and (8) the dispersion

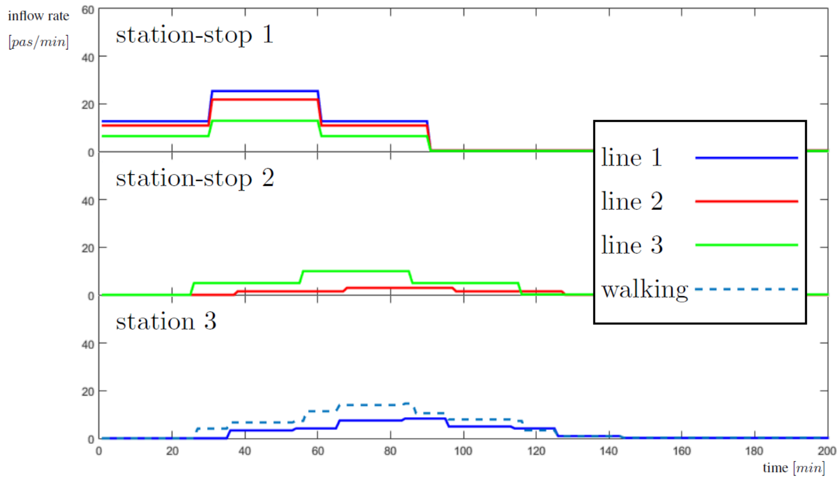

Figure 4: Evolution of inflow rates $[pas/min]$ from each node

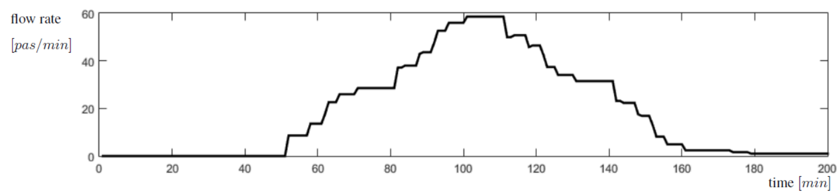

Figure 5: Evolution of flow rates $[pas/min]$ arriving to station 4

parameter for the logit model, θ. Then, being $K = T/\Delta t$ the number of time increments resulting from the discretization, the outputs of the algorithm are the arrays $E = \left(E_{ad}^{k}\right)_{a\in A, d\in D, k=1,...,K}$ and $C = \left(C_{ad}^{k}\right)_{a\in A, d\in D, k=1,...,K}$, where, for an arc $a \in A$, a destination $d \in D$ and a time increment $k \in \{1, ..., K\}$, E_{ad}^{k} is the inflow rate going to destination d entering arc a at time k and C_{ad}^{k} is the cost of using arc c to go to destination d entering at k. The MDTrA algorithm works as follows:

- **Initialization:** Sets of reasonable arcs considering non-overcrowded networks, discrete versions of demand and supply profiles along with other initial values are set.
- **Iterative process:** For each time increment k (increasingly), we apply the following steps:
  - **Exptected minimum costs (Backwards):** Considering the costs, it computes the expected minmum costs, from each destination node and backwards;
  - **Assignment (Forwards):** Considering the expected minimum costs, given a destination d, the flow at each i going to d is split among its outgoing reasonable arcs (to d). After traversing each arc, the inflow is considered as outflow;
  - **Costs update:** For each arc, considering its inflow and time, its costs is updated;
  - **Stop criteria checking:** If the last time increment, K, is the current one, or if there are no more flows to assing in the whole network, then the algorithm ends.

A technically detailed pseudocode for the MDTrA algorithm is presented in Appendix A.3.

5. SMARTCARD DATA USE

The analysis for this study uses Santiago's (Chile) multimodal public transport network, operated by headway scheduling. Data from the fare system is fully integrated, with flat fares between urban buses, metro, and some rail services. In a typical week, 3 million passengers use the system, making 25.5 million trips. The network includes 7 metro lines, more than 300 bidirectional transit lines, and one rail service. The available data includes detailed demand and supply information from three sources: the Automatic Fare Collection (AFC) database, the Automatic Vehicle Location (AVL) database, and the *General Transit Feed Specification* (*GTFS*) database.

The AFC system is implemented through a smartcard, the only payment method, that passengers must validate when boarding a bus or entering a metro station. Even though no alighting validation is needed, the data is processed to estimate boarding and alighting positions using Munizaga & Palma's (2012) methodology. AFC, combined with AVL data, allows identifying the chosen paths between each O-D pair, and to estimate information about trips, such as in-vehicle travel time, out-of-vehicle time (waiting, transfer, and walking times), and number of transfers. Additionally, AVL data from buses (with GPS reporting every 30 $[sec]$) allows obtaining observed frequencies of transit lines to estimate time intervals bewteen buses and waiting times. GTFS data provides geographic information of stops, and structure and scheduled frequency of all lines.

We remark that Santiago's public transport authorities have successfully implemented the static approach for public transit assignment modeling, but it does not take into account the within-day variable demand. To show this dynamic behavior, we collected smartcard data to obtain the number of transactions per minute made at a metro station in a workday, presented in Figure 6. Note that the plot is highly variable along the day, even within periods such as morning peak, afternoon peak, and off peak. This variation, in addition to the variability of observed frequencies, produces passenger congestion at specific times. Our proposed MDTrA model allows to address, simultaneously, this type of dynamic dimension and the uncertainty in passengers choices.

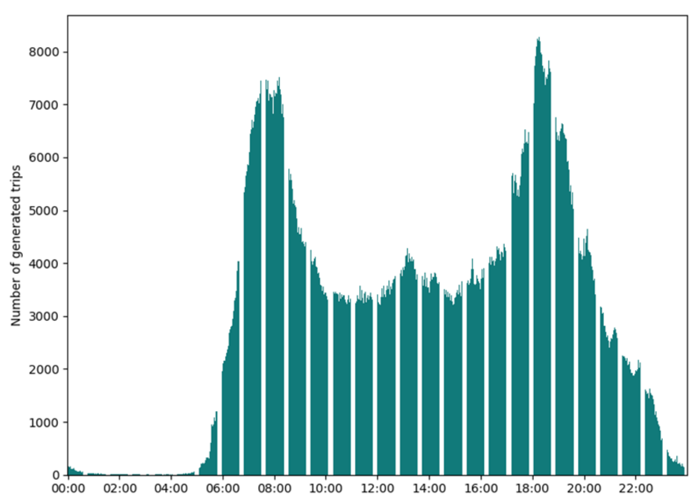

Figure 6: Number of trips generated per minute in a workday (08 May, 2019)

6. RESULTS ON SUBNETWORKS OF SANTIAGO'S PUBLIC TRANSPORT SYSTEM

In this section we present two results worth highlighting, given the overiew that they provide on our approach while still being practical for illustration.

6.1. Reduction of options due to reasonable arcs consideration

Consider Santiago's public transport network and, particularly, stop PG1583 as the destination. Its associated subnetwork has 38 services and 1035 stops (including PG1583). Then, the underlying digraph is subsequently compossed by 1035 nodes and 1648 arcs.

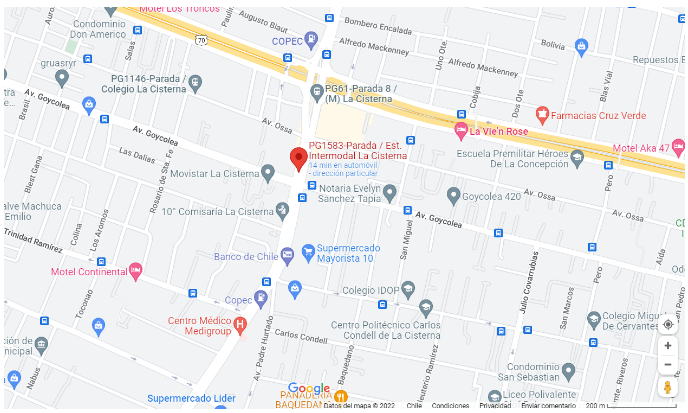

Figure 7: Bus stop PG1583

Given the application of the reasonability concepto, the number of arcs that are actually considered to be assigned with positive flow is reduced to 1084 arcs. This means that more than 34 % of the original arcs won't be chosen, as they are not considered to convinent.

6.2. Overview on the outputs of the MDTrA algorithm

Consider the subnetwork of Santiago, Chile, shown in Figure 8. Here, stop PA215 is the single destination and the rest of the nodes and arcs are defined by the lines that serve at PA215, plus a connection to metro station *La Moneda* (due to their closeness), and their respective stops. The network consists then in 1072 stops and 1055 aggregate trips between stops for all serivices. We use as demand the estimation shown in Figure 6.

To determine times we consider the information provided by ADATRAP to obtain travel times between consecutive stops (i, j), $(V_{(i,j)})$ and average frequences of a line l at each stop $\phi_l(t)$ (constant). Access time (b_a) and exit time (l_a) are set as 0.5 min for all bus stops and 2 min for all metro stations. Walking times for walkable arcs (w_a) are obtained from Google Maps. With this, the underlying digraph of reasonable arcs, is presented in Figure 9.

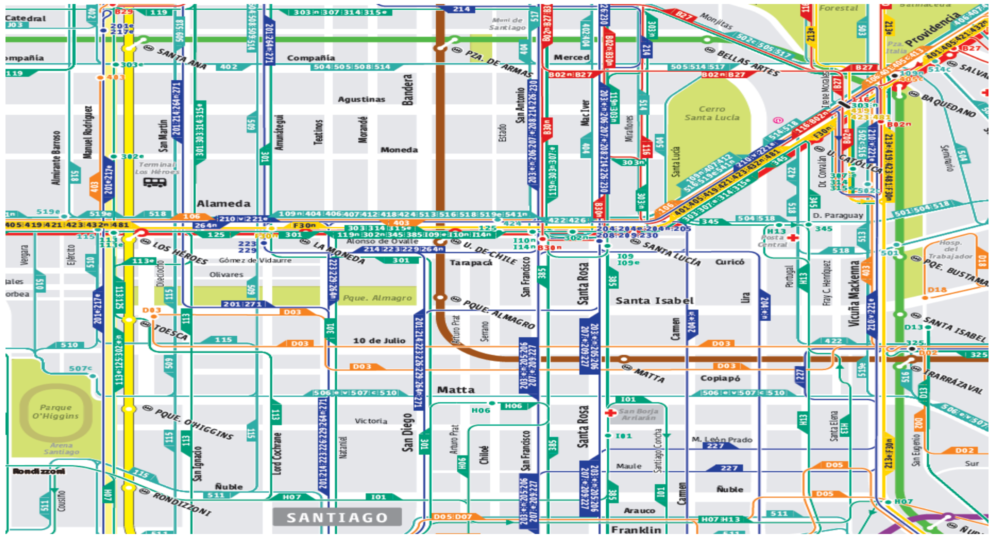

Figure 8: Subnetwork of Santiago's public transport system

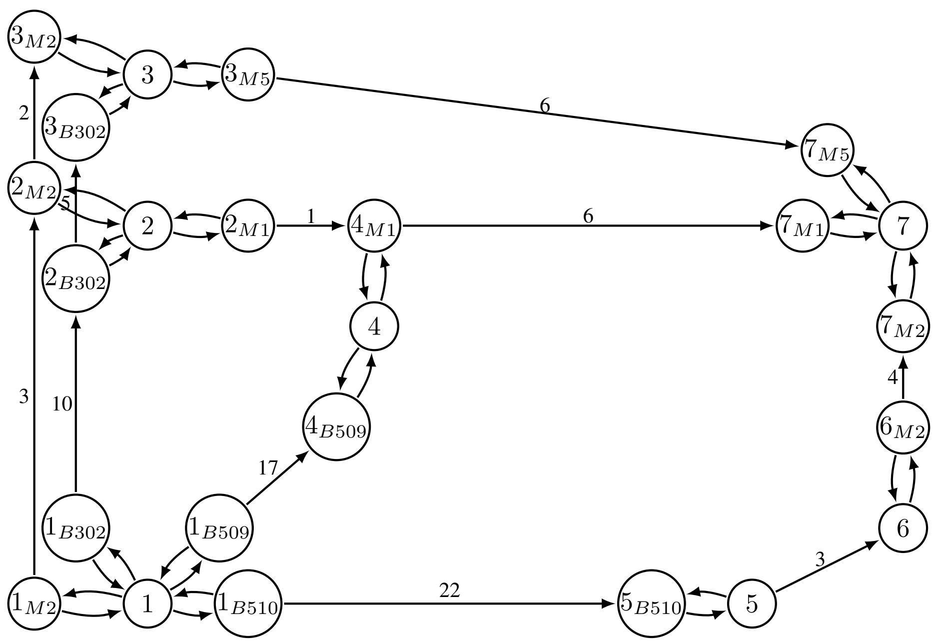

Figure 9: Subnetwork's underlying digraph of reasonable arcs

Costs are represented as *in vehicle time* (*IVT*). The factors to compute costs in Equation (2) (overcrowding is considered 0 zero), come from Tiznado-Aitken et al. (2021), where they elaborated parameters to express different aspects of cost in terms of IVT, as shown in Table 4.

Our computational implementation delivers as results the plots presented in Figures 10, 11 and 12, corresponding to assignments: from Parque O'Higgins metro station to bus linnes B505 and B506 and metro line L2; from metro station Los hores to metro Lines 1 and 2; from Irrarázabal metro station to metro line 5 and to walking option.

Attribute	Units	IVT equivalency	Value
Walking time	min	α,δ,ρ	2
Waiting time	min	β	
Transfers	Direct quantity	p_a	10.2

Table 4: Multipliers to IVT, Tiznado-Aitken et al. (2021)

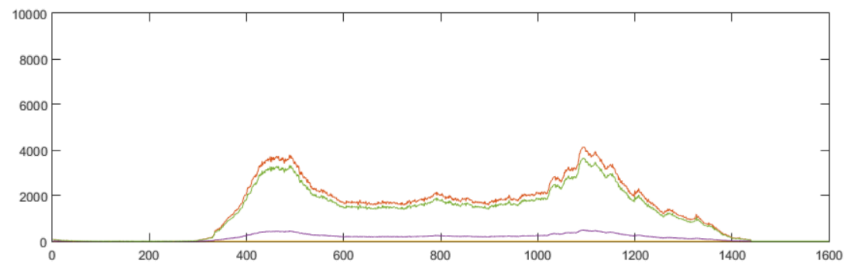

Figure 10: Inflow $[pas/min]$ to B506 (green), L2 (red) and B505 (purple) from Parque O'higgins

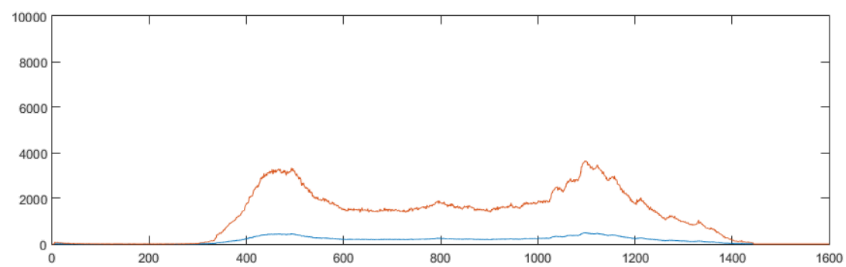

Figure 11: Inflow $[pas/min]$ to L1 (red) and L2 (blue) from Los Héroes

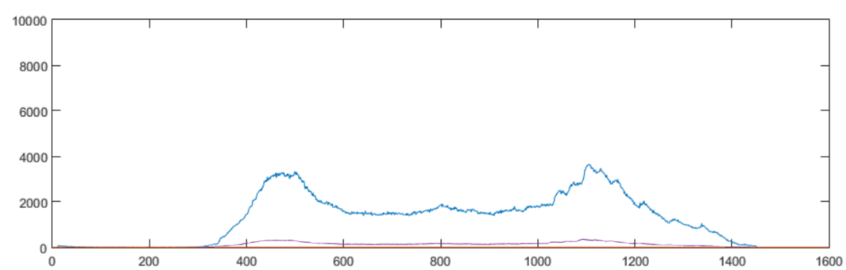

Figure 12: Inflow $[pas/min]$ to L5 (blue) and walking (purple) from Irarrázabal

7. FINAL COMMENTS AND CONCLUSIONS

In this paper, we present a new approach to address the problem of dynamic transit assignment with uncertainty in passenger's choices. We do so by presenting the Markovian dynamic transit assignment (MDTrA) model, showing an example of how the MDTrA model works, giving an insight in how to use smartcard data to obtain the required exogenous demand profile, and proposing the MDTrA algorithm, a solution method for a discrete version of the original problem.

The MDTrA model, intuitivelly, seeks to represent that, according to the transit network's underlying digraph and given demand and supply profiles, at each node (station/stop) passengers travel to their destinations by choosing the next reasonable arc (line/walk) to move forward to, given their perceived costs of using said arc to go to their destinations. The demand is given by a time-dependent function that defines the flow rate of passengers from origins to destinations, while the supply is given by the time-dependent frequencies of the transit network services. The arc travel time functions are fundamental, as they allow us to represent the dynamic aspect of the problem, locating in time the different variables of the model, while the arc cost functions represent the different factors that influence the passengers travelling experience. The arc-choice model, after computing the expected minimum costs of arcs, performs the assignment according to a logit model of known dispersion parameter, and, even though paths are not directly chosen, the routes that passenger follow end up being constructed by a recursive arc-choice process.

We also present how smartcard data can be used to generate a demand profile that evolves over time. Transactions when boarding at each station/stop can be collected, recording the time and estimating the destination, following Munizaga & Palma's (2012) methodology.

The proposed MDTrA algorithm allows obtaining a solution for a version of the model considering a discretization of the time period to be analysed. In brief, after an initialization, it runs over each time increment that results from the discretization a backward step to compute expected minimum costs and then a forward step to perform the assignment.

Some of the defining features of our contribution are that: it reduces the set of options through the reasonability concept; in addition to usual modes (metro and buses), it aknowledges that passengers may choose to walk between stations, adding a mode option not usually considered in the literature; given its arc-based construction, routes costs independence and route enumeration are not needed to formulate the model or to construct the algorithm; and smartcard data, among other type of data, allows constructing the exogenous information that the model and the algorithm requires.

In summary, we propose a model and its corresponding solution method to adress from Markovian point of view the dynamics of the decisions of user given temporal dependant context. Our approach can be considered as an alternative to route base models, as it disaggregate the analysis to an arc-level focused one, being able to aim the objectives of studies to more local instances, as it allows to consider the effects of different criteria over decompossed parts of the network. Our approach, given our algorithm and its impementation, also allows us to study how a network with an exising assignment configuration can be induced to one corresponding to a Markovian point of view.

For later stages of this research, among other ideas, we intend to: apply the effective frequency theory in the dynamic context (as in Cortés et al. (2013); Pineda et al. (2016)); integrate with the recursive approach in Cortés et al. (2023); study diferent instances, fictional and real, to test the implementation of the algorithm; address congestion of passengers in station/stops; represent the effect of consecutive transbords.

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REFERENCES

Baillon, J.-B., & Cominetti, R. (2008, Jan 01). Markovian traffic equilibrium. **Mathematical Programming**, 111 (1), 33–56. Retrieved from https://doi.org/10.1007/s10107-006-0076-2 doi: 10.1007/s10107-006-0076-2

Cepeda, M., Cominetti, R., & Florian, M. (2006). A frequency-based assignment model for congested transit networks with strict capacity constraints: characterization and computation of equilibria. **Transportation Research Part B: Methodological**, 40 (6), 437-459. Retrieved from https://www.sciencedirect.com/science/article/pii/S0191261505000743 doi: https://doi.org/10.1016/j.trb.2005.05.006

Cominetti, R., & Correa, J. (2001). Common-lines and passenger assignment in congested transit networks. **Transportation Science**, 35 (3), 250-267. Retrieved from https://doi.org/10.1287/trsc.35.3.250.10154 doi: 10.1287/trsc.35.3.250.10154

Cortés, C. E., Donoso, P., Gutiérrez, L., Herl, D., & Muñoz, D. (2023). A recursive stochastic transit equilibrium model estimated using passive data from santiago, chile. **Transportation Research Part B: Methodological**, 174, 102780. Retrieved from https://www.sciencedirect.com/science/article/pii/S0191261523001030 doi: https://doi.org/10.1016/j.trb.2023.102780

Cortés, C. E., Jara-Moroni, P., Moreno, E., & Pineda, C. (2013). Stochastic transit equilibrium. **Transportation Research Part B: Methodological**, 51, 29-44. Retrieved from https://www.sciencedirect.com/science/article/pii/S0191261513000210 doi: https://doi.org/10.1016/j.trb.2013.02.001

de Cea, J., & Fernández, E. (1993). Transit assignment for congested public transport systems: An equilibrium model. **Transportation Science**, 27 (2), 133-147. Retrieved from https://doi.org/10.1287/trsc.27.2.133 doi: 10.1287/trsc.27.2.133

de la Paz Guala, R. (2020). **Mdta: Markovian dynamic traffic assignment, a new approach for stochastic dta** (Unpublished doctoral dissertation). Universidad de Chile.

Dial, R. B. (1971). A probabilistic multipath traffic assignment model which obviates path enumeration. **Transportation research**, 5 (2), 83–111.

Kiencke, U., & Nielsen, L. (2000, nov). Automotive control systems: For engine, driveline, and vehicle. **Measurement Science and Technology**, 11 (12), 1828–1828. Retrieved from https://doi.org/10.1088/0957-0233/11/12/708 doi: 10.1088/0957-0233/11/12/708

Lam, W., Gao, Z., Chan, K., & Yang, H. (1999). A stochastic user equilibrium assignment model for congested transit networks. **Transportation Research Part B: Methodological**, 33 (5), 351-368. Retrieved from `https://www.sciencedirect.com/science/article/pii/S019126159800040X` doi: https://doi.org/10.1016/S0191-2615(98)00040-X

Munizaga, M. A., & Palma, C. (2012). Estimation of a disaggregate multimodal public transport origin–destination matrix from passive smartcard data from santiago, chile. **Transportation Research Part C: Emerging Technologies**, 24, 9-18. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0968090X12000095` doi: https://doi.org/10.1016/j.trc.2012.01.007

Nguyen, S., & Pallottino, S. (1988). Equilibrium traffic assignment for large scale transit networks. **European Journal of Operational Research**, 37 (2), 176-186. Retrieved from `https://www.sciencedirect.com/science/article/pii/037722178890327X` doi: https://doi.org/10.1016/0377-2217(88)90327-X

Nielsen, O., & Frederiksen, R. (2006, 04). Optimisation of timetable-based, stochastic transit assignment models based on msa. **Annals OR**, 144, 263-285. doi: 10.1007/s10479-006-0012-0

Nuzzolo, A., & Comi, A. (2017). Transit travel strategy as solution of a markov decision problem: Theory and applications. **2017 5th IEEE International Conference on Models and Technologies for Intelligent Transportation Systems (MT-ITS)**, 850-855.

Pineda, C., Cortés, C. E., Jara-Moroni, P., & Moreno, E. (2016). Integrated traffic-transit stochastic equilibrium model with park-and-ride facilities. **Transportation Research Part C: Emerging Technologies**, 71, 86-107. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0968090X16300961` doi: https://doi.org/10.1016/j.trc.2016.06.021

Spiess, H., & Florian, M. (1989). Optimal strategies: A new assignment model for transit networks. **Transportation Research Part B: Methodological**, 23 (2), 83-102. Retrieved from `https://www.sciencedirect.com/science/article/pii/0191261589900349` doi: https://doi.org/10.1016/0191-2615(89)90034-9

Tavassoli, A., Mesbah, M., & Hickman, M. (2020, 10). Calibrating a transit assignment model using smart card data in a large-scale multi-modal transit network. **Transportation**, 47. doi: 10.1007/s11116-019-10004-y

Tiznado-Aitken, I., Muñoz, J. C., & Hurtubia, R. (2021). Public transport accessibility accounting for level of service and competition for urban opportunities: An equity analysis for education in santiago de chile. **Journal of Transport Geography**, 90, 102919. Retrieved from `https://www.sciencedirect.com/science/article/pii/S0966692320309960` doi: https://doi.org/10.1016/j.jtrangeo.2020.102919

Yang, L., & Lam, W. H. K. (2006). Probit-type reliability-based transit network assignment. **Transportation Research Record**, 1977 (1), 154-163. Retrieved from `https://doi.org/10.1177/0361198106197700118` doi: 10.1177/0361198106197700118

A. PSEUDOCODES FOR ALGORITHMS

A.1. Merging of stops

Algorithm 1 (CN, CA) = NetworkCollapser$((N, A), r)$

$CN \leftarrow \emptyset$
$CA \leftarrow \emptyset$
$N_0 \leftarrow N$
for $i = 1, ..., length(N_0)$ **do**
    for all $j \in B(i, r)$ **do**
        $CN \leftarrow CN \bigcup \{i_c\}$
        $CA \leftarrow CA \bigcup \{(i_c, k) : (j, k) \in A\}$
        $N_0 \leftarrow N_0 - \{j\}$
    end for
    $N_0 \leftarrow N_0 - \{i\}$
end for

A.2. Transit network underlying digraph construction

Algorithm 2 (N, A) = NetworkBuilder(DB)

$N \leftarrow \emptyset$
$A \leftarrow \emptyset$
for i=1,...,length(DB) **do**
    $serv.i \leftarrow struct2cell(DB(i))$
    $N \leftarrow N \bigcup serv.i\{par\}$
    $A_i \leftarrow \emptyset$
    for j=1,...,length(serv.i{ par })-1 **do**
        $A \leftarrow A \bigcup \{(serv.i\{par\}(j), serv.i\{par\}(j+1)\}$
    end for
end for

A.3. MDTrA algorithm

Algorithm 3 (E, C) = MDTrA$\left((N, A), D, (\mathcal{D}(\cdot), \phi(\cdot)), P, V, T, \Delta t, \theta\right)$

STEP 0: INITIALIZATION Technical settings
for k=1,...,K **do**
  STEP 1: BACKWARD
  for all $d \in D$ **do**
    for all $i \in N$, in the order given π_d **do**
      for all $a = (i, j) \in A_i^-$ incoming arcs to i, **do**
        Compute expected minimum costs of using a to go to d
      end for
      Compute expected minimum costs from i to d
    end for
  end for
  STEP2: FORWARD
  for all $i \in N$ **do**
    for all $d \in D$ **do**
      for all $a = (i, j) \in A_i^+$ outgoing arcs from i, **do**
        Assign the aggregation of outflow rates of incoming arcs to i, except by (j, i), and the flow rate generated at i as inflow rate
      end for
    end for
  end for
  STEP 3: COST UPDATES
  for all $a \in A$ **do**
    Update the cost of a
  end for
  STEP 4: STOP CONDITION
  if $k = K$ or there are no more flow rates to assign **then**
    End MDTrA algorithm
  end if
end for