# A Markovian approach for dynamic transit assignment: Results in subnetworks of Santiago

Ricardo de la Paz Guala[1]
Marcela Munizaga[1]
Jaqueline Arriagada[2]

[1]División Transporte, Departamento de Ing. Civil, Universidad de Chile, Santiago, Chile
[2]University of Leeds, Leeds, Reino Unido

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# The MDTrA Approach

## Overview

- It deals with time-dependent demand and supply and uncertainty in passengers' route-choices
- Adapts the *Markovian Traffic Equilibrium* concept (Baillon and Cominetti, 2008) to a dynamic and stochastic conext (Addison and Heydecker, 1996, 1998; Heydecker and Addison, 2005; Han, 2003; Heydecker and Addison, 1997)
- Integrates a *reasonability* concept
- It is arc-based, rather than route-based
- The route-choice results from recursive arc-choices

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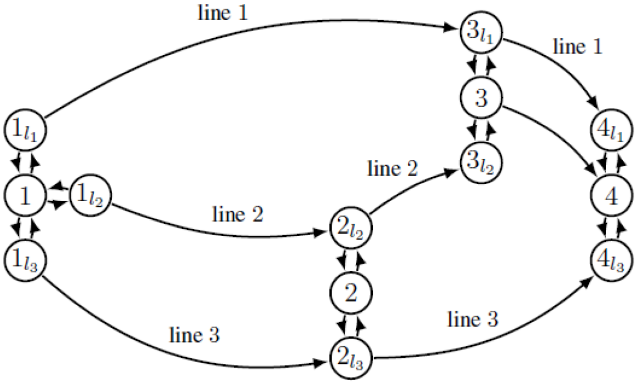
# Underlying digraph

## A representation of the options



# Travel is focused on destinations

**Definition**

Given a destination node $d$, an arc $(i, j)$ will be a *reasonable arc towards* $d$ if the minimum cost of going from $j$ to $d$ is not greater than the minimum cost of going from $i$ to $d$.

The set of all reasonable arcs towards $d$ is denoted $R_d$

# Travel is focused on destinations

## Definition

Given a destination node $d$, an arc $(i, j)$ will be a *reasonable arc towards* $d$ if the minimum cost of going from $j$ to $d$ is not greater than the minimum cost of going from $i$ to $d$.
The set of all reasonable arcs towards $d$ is denoted $R_d$

## Assumption

Motorists only travel through reasonable arcs to their destinations

# The MDTrA model

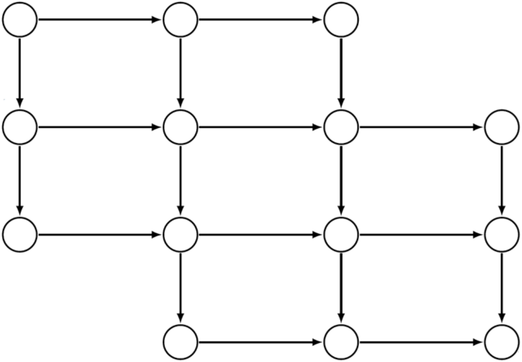
## Overview

- It is composed by *demand and supply profiles*, a *time and cost functionsl* and an *arc-choice model*
- The demand and supply profiles are exogenous
- The time and cost functions classify arcs by type
- The arc choice model is a dynamic version of the one associated with the MTE

# Transport network

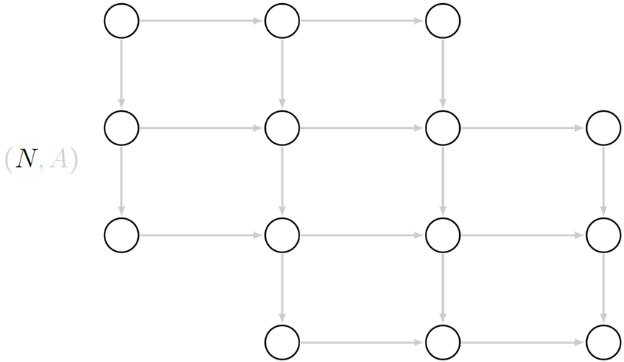
## The digraph

$(N, A)$



# Transport network

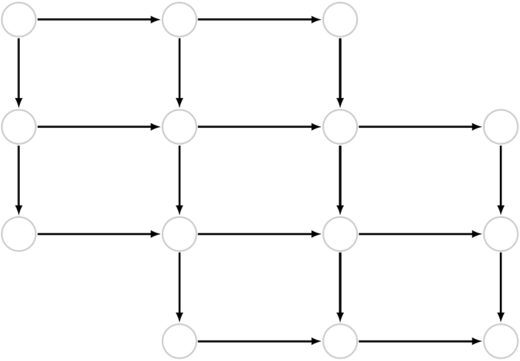
## The set of nodes



# Transport network

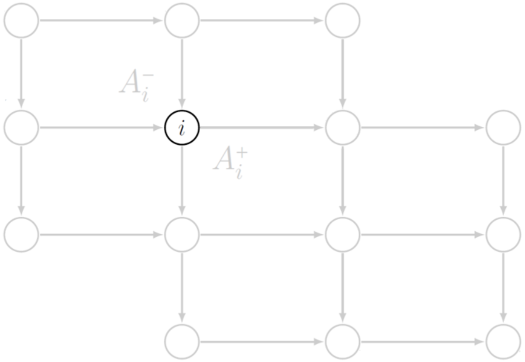
## The set of arcs

$(N, A)$



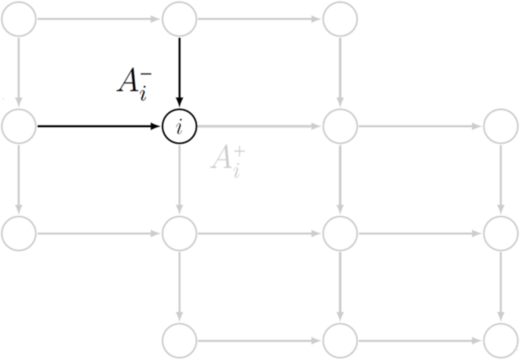
# Sets associated with a node

**$i \in N$**



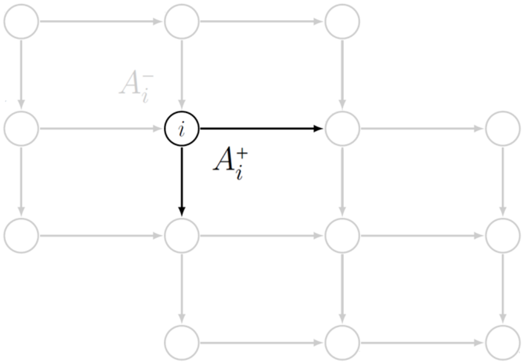
# Sets associated with a node

## Set of incoming arcs to a node, $A_i^-$



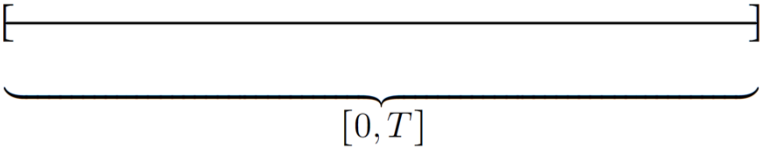
# Sets associated with a node

## Set of outgoing arcs from a node, $A_i^+$



# Analyzed time period

## The time interval



# The first structure

The demand and supply profiles

# The demand and supply profiles

## The demand profile

For each $(o, d)$ of the transport network, the demand rate function from the origin $o$ to the destination $d$, denoted as $\mathcal{D}_{(o,d)}(\cdot)$, is exogenous information

## The supply profile

The supply rate function is represented through the frequency function of each service at each stop, and is exogenous information

## The second structure

# The time and cost functions

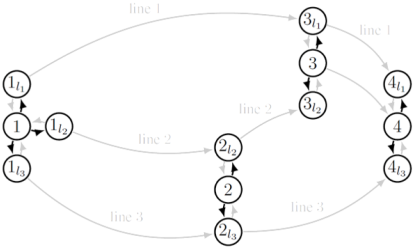
# The time and cost functions

## Overview

- Represents the interactions within each arc $a = (i, j)$
- Relates the inflow and outflow rates going to each destination
- Locate the flows temporarily
- Respects FIFO rule

# The time and cost functions

$\forall d \in D, \forall a \in A, \forall t \in [0, T]$

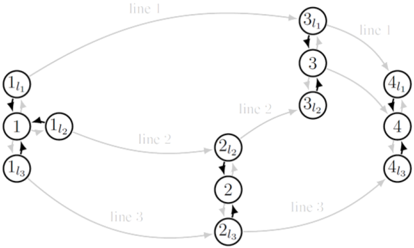



$$T_a(t) = b_a + \frac{1}{\phi_a(b_a+t)}$$

$$C_{ad}(t) = \alpha b_a + \frac{\beta}{\phi_a(b_a+t)}$$

# The time and cost functions

$\forall d \in D, \forall a \in A, \forall t \in [0, T]$

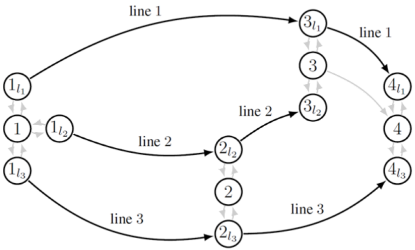



$$T_a(t) = l_a$$

$$C_{ad}(t) = \begin{cases} \delta l_a, & \text{if } a \in A_a \text{ and } j = d \\ \delta l_a + p_a, & \text{if } a \in A_a \text{ and } j \neq d \end{cases}$$

# The time and cost functions

$\forall d \in D, \forall a \in A, \forall t \in [0, T]$

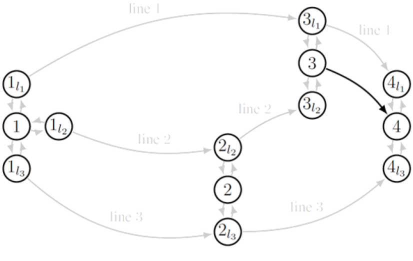



$$T_a(t) = V_a$$

$$C_{ad}(t) = \gamma V_a + H_a \sum_{d' \in D} E_a^{d'}(t)$$

# The time and cost functions

$\forall d \in D, \forall a \in A, \forall t \in [0, T]$




$$T_a(t) = w_a$$

$$C_{ad}(t) = \rho w_a$$

# The third structure

The arc-choice model

# The arc-choice model

## Overview

- Extends to a dynamic assignment the one associated with the MTE considering a logit model
- First, computes recursively the expected minimum costs to each destination
- Then, aggregates flow rates arriving to each node
- Finally, assigns the inflow rates to the outgoing arcs of each node

# The arc-choice model

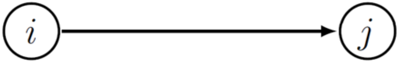
## Logit model specifications

The *dispersion parameter* $\theta$ is considered known

# The arc-choice model: Expected minimum costs

$$\forall d \in D, \forall a = (i,j) \in A, \forall t \in [0, T]$$

$$a = (i,j)$$

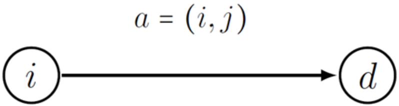



$$W_{jd}(t + T_a(t)) = 0$$

$$Z_{ad}(t) = C_{ad}(t)$$

# The arc-choice model: Expected minimum costs

**If $j = d$**




$$W_{dd}\left(t + T_a(t)\right) = 0$$

$$Z_{ad}(t) = C_{ad}(t)$$

# The arc-choice model: Expected minimum costs

**Expected minimum cost from $j$ to $d$ at $t + C_a(t)$, $W_{jd}(t + C_a(t))$**

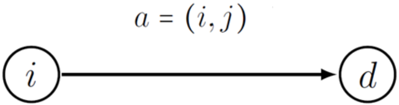
$$a = (i, j)$$


$$W_{dd}(t + T_a(t)) = 0$$

$$Z_{ad}(t) = C_{ad}(t)$$

# The arc-choice model: Expected minimum costs

**Expected minimum cost of using arc *a* to go to *d* at *t*, $Z_{ad}(t)$**

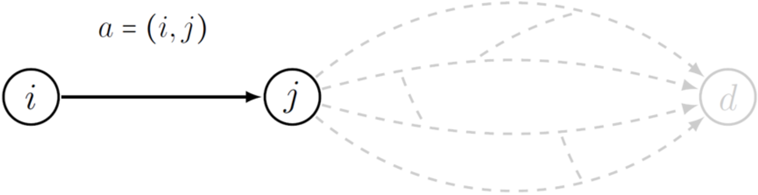



$$W_{dd}(t + T_a(t)) = 0$$

$$Z_{ad}(t) = C_{ad}(t)$$

# The arc-choice model: Expected minimum costs

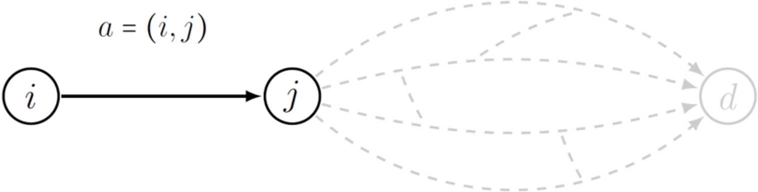
## If $j \neq d$




$$Z_{ad}(t) = C_{ad}(t) - \frac{1}{\theta} \ln \left( \sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}(t + T_b(t))\right) \right)$$

# The arc-choice model: Expected minimum costs

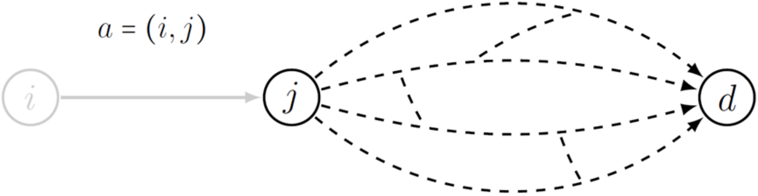
## $Z_{ad}(t)$ components: Cost of the arc $a$ at $t$




$$Z_{ad}(t) = C_{ad}(t) - \frac{1}{\theta} \ln \left( \sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}(t + C_b(t))\right) \right)$$

# The arc-choice model: Expected minimum costs

## $Z_{ad}(t)$ components: Expected minimum cost from $j$ to $d$




$$Z_{ad}(t) = C_{ad}(t) - \frac{1}{\theta} \ln \left( \sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}(t + T_b(t))\right) \right)$$

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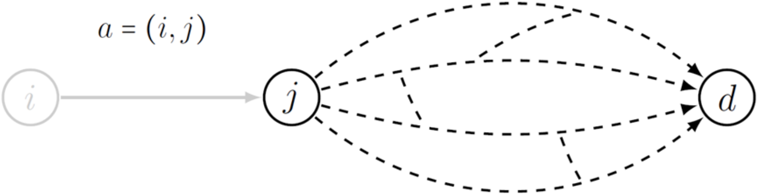
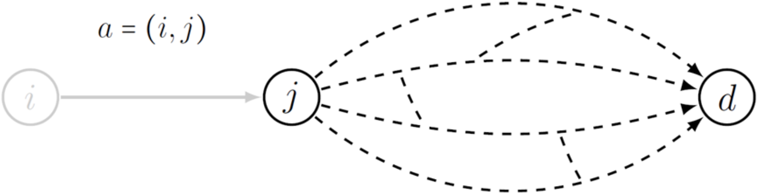



$$Z_{ad}(t) = C_{ad}(t) - \frac{1}{\theta} \ln \left( \sum_{b \in A_j^+} \exp\left(-\theta Z_{bd}(t + T_a(t))\right) \right)$$

# The arc-choice model: Expected minimum costs

## $Z_{ad}(t)$ components: Expected minimum cost from $j$ to $d$

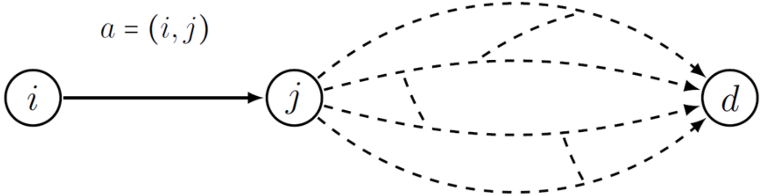
$a = (i, j)$




$$Z_{ad}(t) = C_{ad}(t) + W_{jd}(t + T_a(t))$$

# The arc-choice model: Expected minimum costs

**Expected minimum cost of using arc *a* to go to *d* at *t***




$$Z_{ad}(t) = C_{ad}(t) + W_{jd}\left(t + T_a(t)\right)$$

# The arc-choice model: Expected minimum costs

$\forall d \in D, \forall a = (i,j) \in A, \forall t \in [0, T]$

If $j = d$, then

$$W_{jd}(t + T_a(t)) = 0 \quad (1)$$

$$Z_{ad}(t) = C_a(t) \quad (2)$$

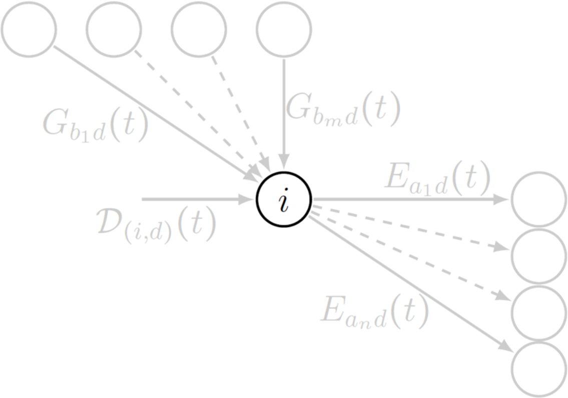
otherwise, if $j \neq d$

$$W_{jd}(t + T_a(t)) = -\frac{1}{\theta} \ln \left( \sum_{b \in A_j^+} \exp\left(-\theta \left(Z_{bd}\left(t + C_a\left(t\right)\right)\right)\right) \right) \quad (3)$$

$$Z_{ad}(t) = C_{ad}(t) + W_{jd}(t + C_a(t)) \quad (4)$$

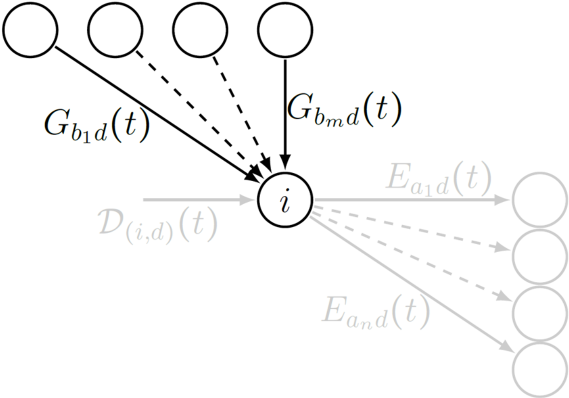
# The arc-choice model: Assignment

**Given a destination node $d \in D$, $\forall i \in N$**



# The arc-choice model: Assignment

$\forall d \in D, \forall t \in [0, T]$



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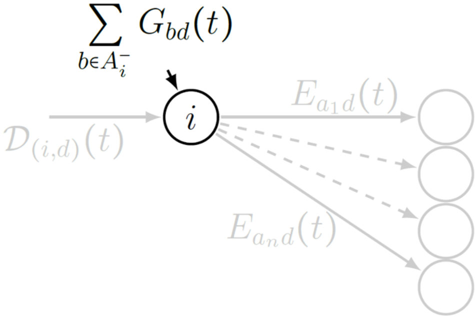
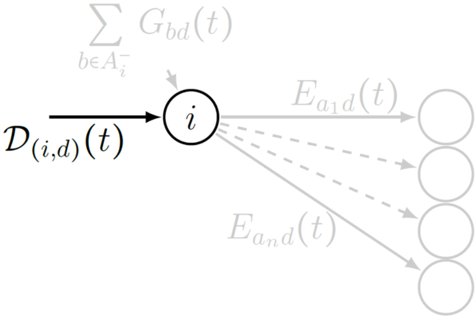


# The arc-choice model: Assignment

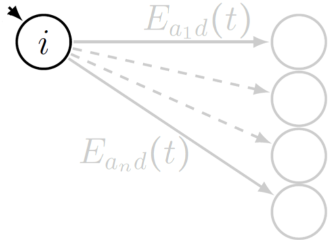
$\forall d \in D, \forall t \in [0, T]$



# The arc-choice model: Assignment

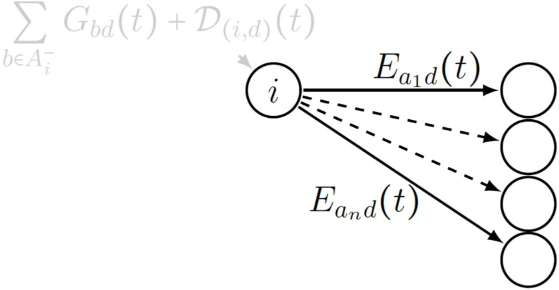
$\forall d \in D, \forall t \in [0, T]$

$$\sum_{b \in A_i^-} G_{bd}(t) + \mathcal{D}_{(i,d)}(t)$$



# The arc-choice model: Assignment

$\forall d \in D, \forall a \in A_i^+, \forall t \in [0, T]$



# The arc-choice model: Assignment

$\forall d \in D, \forall i \in N, \forall a \in A_i^+, \forall t \in [0, T]$

$$E_{ad}(t) = \frac{\exp\left(-\theta Z_{ad}\left(t\right)\right)}{\sum\limits_{b \in A_i^+ \bigcap R_d} \exp\left(-\theta Z_{bd}\left(t\right)\right)} \left( \sum_{b \in A_i^-} G_{bd}(t) + \mathcal{D}_{(i,d)}(t) \right) \quad (5)$$

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# The MDTrA algorithm

## Overview

- Works over a discretization of the analyzed time period
- The processes at each time increment are based on a reversed *Dial's Algorithm*

# Overview

## The inputs

- $(N, A)$, digraph associated to the transport network
- $O \subseteq N$, set of origins
- $D \subseteq N$, set of destinations
- $OD \subseteq O \times D$, set of $O - D$ pairs
- $\mathcal{D}(\cdot)$, time-dependent demand rate
- $T$, length of the period of time to analyze
- $\theta$, dispersion parameter

# Overview

## The inputs

- $(N, A)$, digraph associated to the transport network
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- $OD \subseteq O \times D$, set of $O - D$ pairs
- $\mathcal{D}(\cdot)$, time-dependent demand rate
- $T$, length of the period of time to analyze
- $\theta$, dispersion parameter
- $\Delta t$, timestep size

# Overview

## The outputs

The discretization will result in $K$ time intervals. Then, $\forall d \in D$, $\forall a \in A$, $\forall k = 1, \dots, K$

- $E_{ad}^k$, inflow rate of arc $a$ going to $d$ at $k$
- $G_{ad}^k$, outflow rate of arc $a$ going to $d$ at $k$
- $C_{ad}^k$, cost of arc $a$ going to $d$ at $k$
- $Z_{ad}^k$, expected minimum cost from arc $a$ to $d$ at $k$
- $W_{id}^k$, expected minimum cost from node $i$ to $d$ at $k$

# How it works

## Technical settings

- STEP 0: Initialization

## From $k = 1$ to $k = K$ or until meeting stop criteria

- STEP 1: **Backward**: Computes expected minimum costs
- STEP 2: Computes assignment factors (logit model coefficients)
- STEP 3: **Forward**: Assigns inflow rates to each destination
- STEP 4: Updates costs of arcs
- STEP 5: Checks if stop criteria are meet

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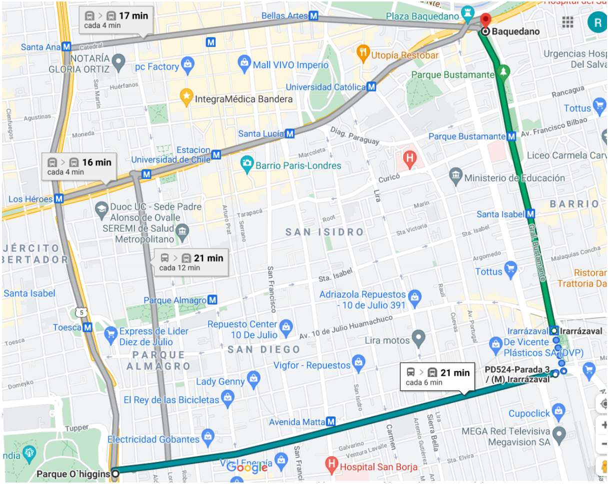
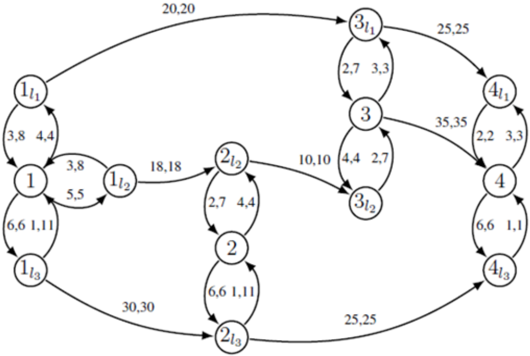
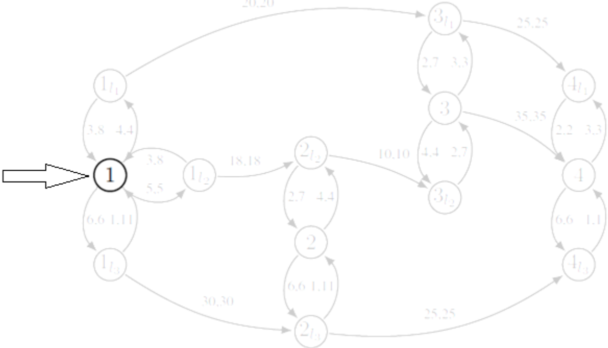
# Illustrative case

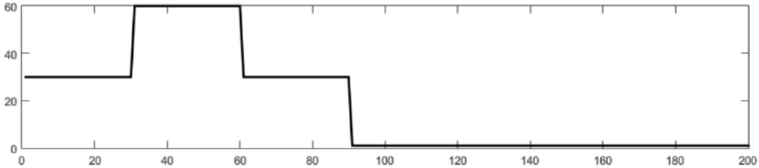


# Illustrative case

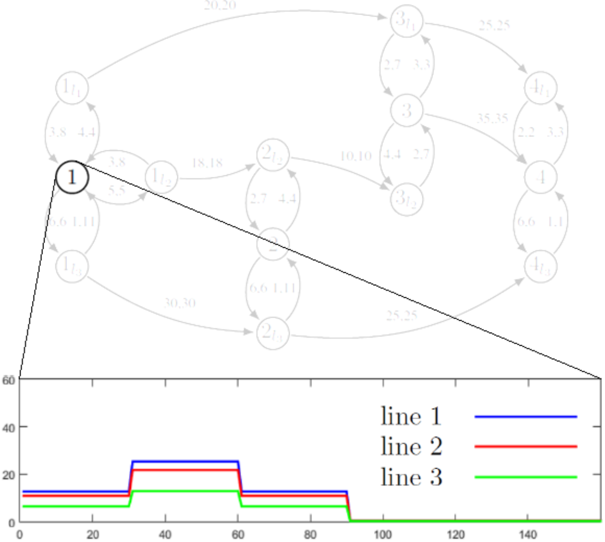


# Illustrative case



# Illustrative case



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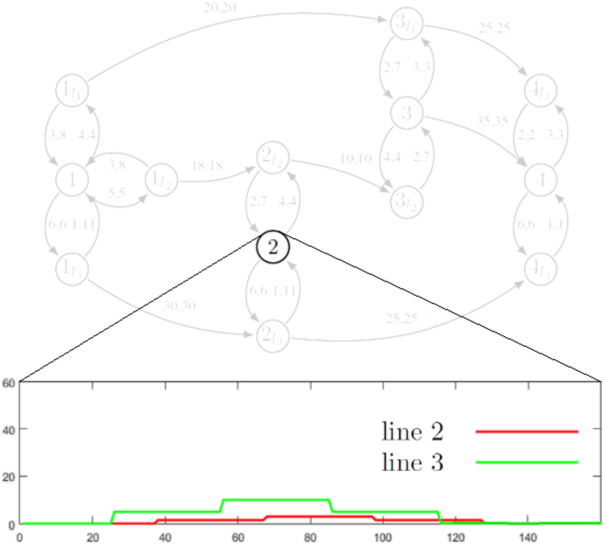

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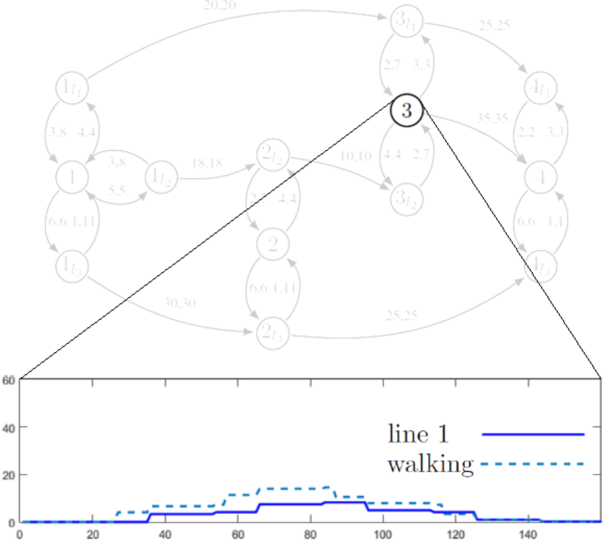

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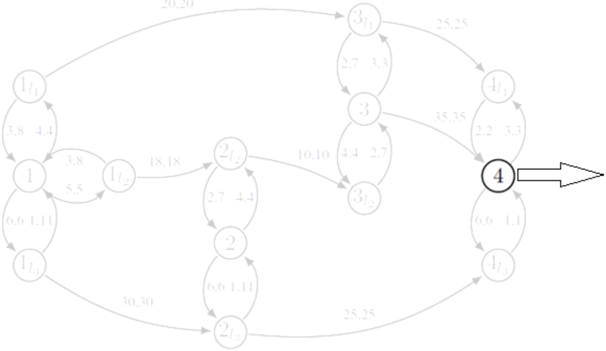
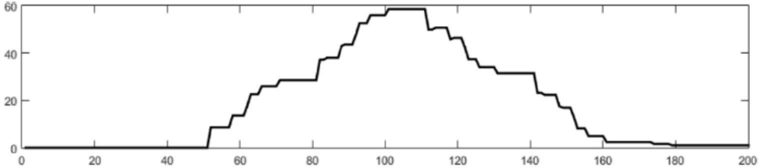
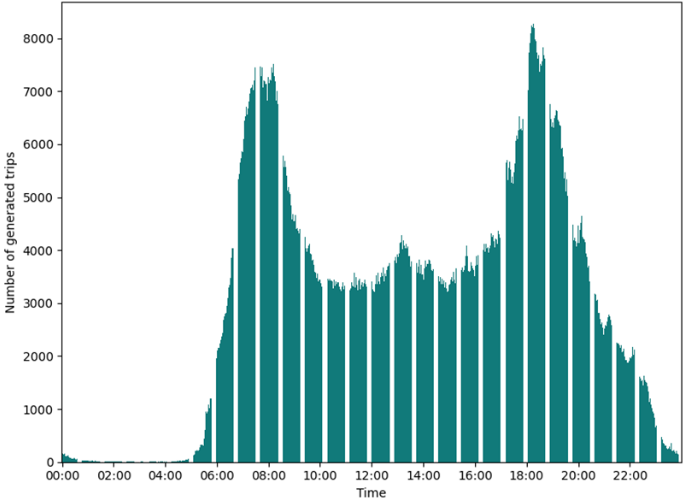
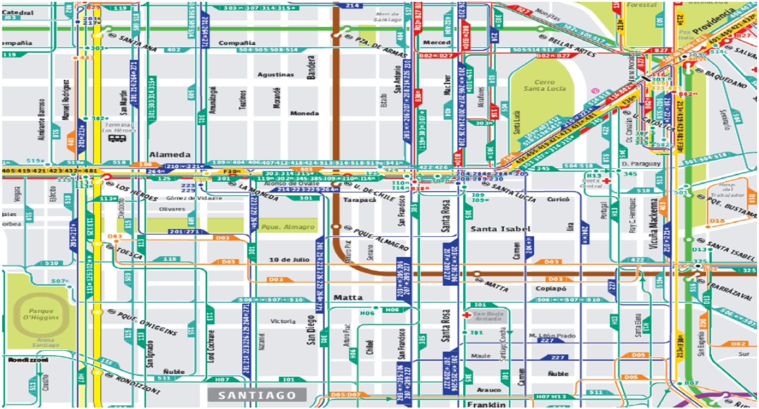


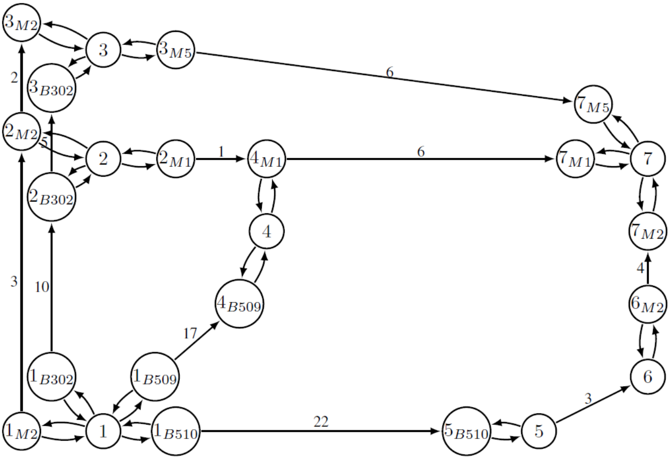
# Real data: Demand estimation with smartcard data



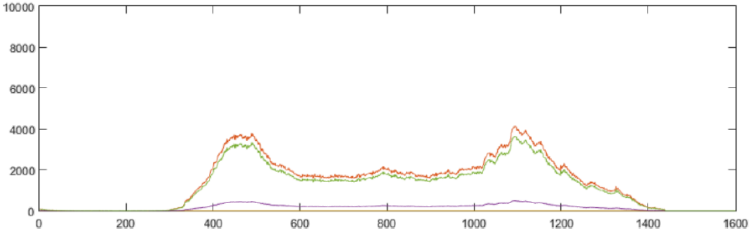
# Real data: Subnetwork A of Santiago



# Real data: Reasonable arcs for subnetwork A

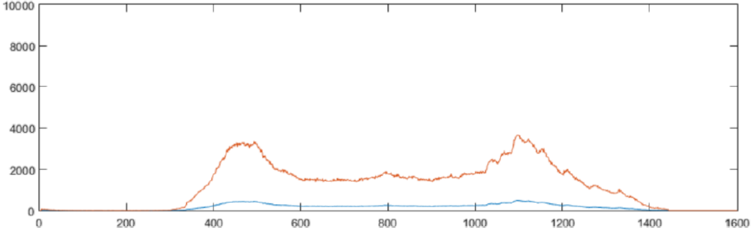


# Real data: Inflows from Parque Ohiggins



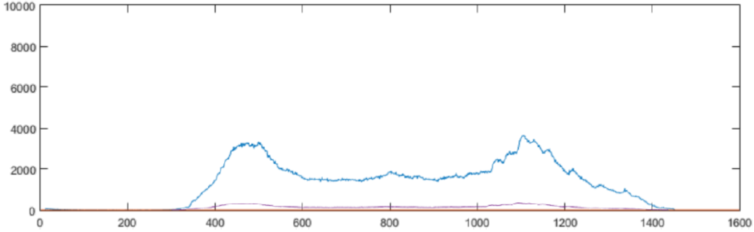

B506 (green), L2 (red) and B505 (purple) from Parque O'higgins

# Real data: Inflows from Los Héroes



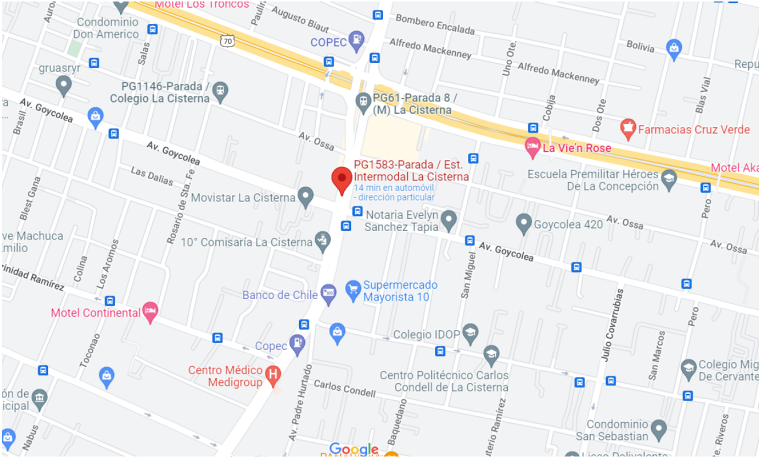

L1 (red) and L2 (blue) from Los Héroes

# Real data: Inflows from Irarrázabal




L5 (blue) and walking (purple) from Irarrázabal

# Real data: Subnetwork B of Santiago



# Real data: Subnetwork B of Santiago

## Characteristics

- Intermodal La Cisterna as the single destination
- Associated with 38 services
- Underlying digraph originally has 1035 nodes and 1648 arcs

# Real data: Subnetwork B of Santiago

## Characteristics

- Intermodal La Cisterna as the single destination
- Associated with 38 services
- Underlying digraph originally has 1035 nodes and 1648 arcs

## After applying reasonability concept

- 1084 arcs
- More than 34 % of reductions of arcs

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# Features

## The MDTrA model

- Adapts an MTE's arc-choice model to a dynamic and stochastic assignment context
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## The MDTrA algorithm

- Does not require routes enumeration
- Allows initialization with non-empty networks
- Efficiently runs network loading though dynamic programming

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# Future works

The present work and its results open research opportunities on:

- Extend the reasonability concept
- Study initialization with non-empty networks
- Use the algorithms' outputs as initial solution and then apply improvement methods

# Thank you for your attention

Questions? feedback? Please, reach us

ricardodelapaz@uchile.cl

dic INGENIERÍA CIVIL UNIVERSIDAD DE CHILE

ISCI INSTITUTO SISTEMAS COMPLEJOS DE INGENIERÍA

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